

Minimization of Energy Consumption Per Bit of Wireless Networked Control Systems with Time-Correlated Nakagami- m Fading Channels

Rafaela Scaciota, Glauber Brante, Guilherme Luiz Moritz and Richard Demo Souza

Abstract—In this paper we study the energy consumption of wireless networked control systems (WNCSs) with time correlation. Our proposal is to minimize the energy consumption, assuming that the actuators may be battery-powered devices, without affecting the control system stability, which may be impacted by communication errors. Firstly, the communication channel is modeled according to a time-correlated Nakagami- m fading. With the outage probability for this scenario in hand, we derived a closed-form expression for the transmission power that minimizes the amount of energy consumed per bit in this WNCS scenario. Our results show that the energy consumption increases with the correlation. This is due to the fact that stability depends on the number of consecutive outage events, so that time-correlation implies in higher outage probability, increasing the transmit power in order to maintain the system stable.

Keywords—Wireless networked control systems, time-correlation, consecutive outage events, energy consumption.

I. INTRODUCTION

In recent years, an industrial revolution known as Industry 4.0 has been emerging. One of the main characteristics of this concept is that communication abilities and real-time interaction between operators, products and machines become essential to the production [1]. In this scenario, 5G networks are the key to provide the necessary communication infrastructure. Nevertheless, the requirements in terms of latency, reliability and coverage, denoted as ultra-reliable and low-latency communications (URLLC), impose important technological challenges to the 5G network [2].

Wireless networked control systems (WNCSs) are important examples of the Industry 4.0, in which a closed-loop control system is established using wireless links between actuators, sensors and controllers [3]. Such approach allows quick-deployment and reconfigurability inside the factory, allowing a wide range of automations [4], but it comes at the cost of the unreliable nature of the wireless channel, which may insert packet losses and delays, jeopardizing the system stability. From the one hand, low outage probability at the communication system is required to improve the control system stability, which comes at the expense of higher transmit power.

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On the other hand, a setup for quick deployment inside the plant implies in battery-operated devices, for which low energy consumption is essential to maximize the devices lifetime.

In the literature, the interaction between networks and control systems has been studied, e.g., in [5], where the authors present a cross-layer optimization of a WNCS. The concept of co-design is explored in order to jointly conceive the wireless network and the controller. In this case, the co-design is based on a constrained optimization problem whose objective function is the energy consumption of the network, while the constraints are the packet loss probability and the delay.

A co-design involving communications and control is also presented in [6] where the the impact of multiple antennas is investigated. In this paper a 8×8 multiple-input multiple-output (MIMO) channel is considered, and the authors study the relationship between the multiplexing and the diversity gains when the channel arrays are not known at the transmitter, but only at the receiver. So, the performance obtained by the MIMO design is better than that with a single antenna in terms of average energy consumption and the cost of controlling the system.

However, the authors in [5], [6] do not analyze the stability of the system in the case of packet dropouts. In this context, in [7] it is proposed an adaptive coded modulation approach to guarantee stability and maximize the spectral efficiency in WNCSs. Three schemes are proposed, with the first maximizing the throughput with energy constraints, the second minimizing the energy consumption with throughput constraints, and the third minimizing the transmission delay with energy constraints. Their results show that each scheme performs better depending on the given channel conditions and constraints.

With the goal of improving the robustness of the controller face to packet dropouts, the authors in [8] employ a predictive packet control scheme, so that the controller defines a sequence of control commands in advance. Then, it is proposed a communication/control co-design method with the goal of optimizing the prediction length of the controller. In addition, the finite-blocklength capacity is employed by [8], since communication packets for WNCSs may be shorter than in usual communication scenarios, due to the typical delay restrictions [9].

Another important connection between control systems and wireless communications is investigated by [10], which used the discrete-time switched systems approach to model the stability of a WNCS. This approach consists in defining

subsystems (with respect to the outage of the links), which can be either stable or unstable. Then, in the scenario investigated by [10] two wireless links are considered, the first between sensor and controller and the second between controller and actuator. In this case, four subsystems are possible: three unstable (when one or both links fail) and only one stable (when both links have no packet loss). Thus, the outage probability dictates which subsystem is active in every instant of time. This allows the authors to define stability conditions as a function of an average dwell-time, defined as the average time between subsystems switches.

Furthermore, in [11] we have extended the concepts of [10] in order to minimize the energy consumption of a WNCS. Our communication/control co-design is defined as an optimization problem, with the objective function being the total energy consumption, restricted to the average dwell-time conditions that ensure stability. As a result, we have provided a closed-form expression for the maximal tolerable outage probability of the communication system and the optimal bit rate, which are then used to minimize the total energy consumption per bit. Compared to the usual approach in the literature, which is to treat communication and control systems independently, fixing the outage probability according to a reliability constraint, the proposal in [11] shows a decreased energy consumption.

However, none of the papers above considered the temporal correlation in the transmission channel, which can considerably impact the design of the WNCS. For example, the performance of the approach used in [11] can be impacted by time-correlation. Time-correlation is relevant in the analysis of the physical system for a URLLC scenario. In this case the transmissions are of short duration, with short intervals between transmissions. With deep time-correlation, several outage events can happen repeatedly, so that the system becomes unstable for a longer period of time, making the control task very difficult.

Thus, in order to provide robust design for these cases, in this paper we investigate the energy consumption when the communication channel is modeled according to a time-correlated Nakagami- m fading. First, we study the outage probability subjected to time-correlation. In particular, we are interested in the case of consecutive outage events, i.e., when multiple outages occur one after the other. Then, we minimize the energy consumed per bit for this WNCS by constraining the consecutive outage probability to a given reliability threshold. We compare the cases of full time-correlation and without time-correlation. The numerical results show that the energy consumption with full time-correlation is 4 times higher compared to the case without time-correlation. Our results also show an exponential increase of the energy consumption with the number of consecutive outages.

The remainder of this paper is organized as follows. The communication system model is detailed in Section II. Section III formulates the outage probability in time-correlated Nakagami- m fading. The optimization problem to minimize the energy consumption is formulated in Section IV, Section V presents a few numerical examples and, finally, Section VI concludes the paper.

II. SYSTEM MODEL

Consider the control system depicted in Figure 1, where there is one wireless communication link between the controller and the actuator while the connection between the sensor and the controller is made by a wired link. We consider that these control elements are clock driven. Moreover, although a single sensor-actuator pair is considered in the analysis, our framework can be extended to more than two wireless links, e.g., the case of a remote controller operating with multiple sensors or multiple actuators.

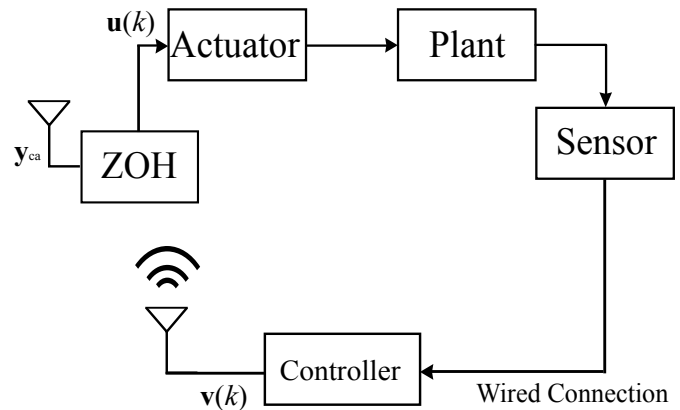


Fig. 1. General architecture of a distributed control system in a wireless communication network.

Then, at each time period k the controller observes the system state, and $\mathbf{v}(k)$ it is the state vector, containing the set of control variables, transmitted to the actuator, which receives

$$\mathbf{y}_{ca}(k) = \sqrt{P_{ca}\kappa_{ca}} h_{ca}(k) \mathbf{v}(k) + \mathbf{n}_{ca}, \quad (1)$$

where P_{ca} is the transmit power used by the controller, κ_{sc} represents the path-loss, $h_{ca}(k)$ represents the quasi-static channel fading, whose envelope follows a time-correlated Nakagami- m distribution, \mathbf{n}_{ca} is the AWGN vector, assumed to have zero mean and variance $\frac{N_0}{2}$ per dimension and N_0 is the unilateral noise power spectral density (psd). The average signal-to-noise ratio (SNR) is given by [12]

$$\bar{\gamma}_{ca} = \frac{P_{ca}G \left(\frac{c}{f}\right)^2}{N_0B(4\pi)^2 d_{ca}^\alpha M}, \quad (2)$$

where c is the speed of light in vacuum, f is the carrier frequency, G represents the combined antenna gains, B is the system bandwidth, d_{ca} is the distance between controller and actuator, α is the path-loss exponent, and M is the link margin. Moreover, the instantaneous SNR is given by [12]

$$\gamma_{ca}(k) = \bar{\gamma}_{ca} h_{ca}^2(k). \quad (3)$$

In the case of an outage, the zero-order hold (ZOH) is responsible for keeping the previous received value, once the system does not use any prediction for the actuator actions. Then, \mathbf{u} is the actuator input signal and depends on the outage between the controller and the actuator. So, the output of the ZOH without transmission errors is $\mathbf{u}(k) = \mathbf{v}(k)$, while the actuator input in the case of an outage event is $\mathbf{u}(k) = \mathbf{u}(k-1)$.

III. OUTAGE PROBABILITY

We now investigate the probability of consecutive outages assuming time-correlation. Let us consider the case of L consecutive outage events, which may happen depending on the system dynamics and time-correlation of the wireless channel, and that the WNCS will have to deal with. We denote the channel envelopes by the set $|\mathbf{h}_L| = \{|h_{ca}(1)|, \dots, |h_{ca}(L)|\}$. Then, following [13], the joint distribution of $|\mathbf{h}_L|$ can be written as a multivariate Nakagami- m distribution with exponential correlation, whose joint probability density function (PDF) is given by [13]

$$f_{|\mathbf{h}_L|}(z_1, \dots, z_L) = \int_{t=0}^{\infty} \frac{t^{m-1}}{\Gamma(m)} e^{-t} \times \prod_{l=1}^L \frac{2z_l^{2m-1}}{\Gamma(m) \left(\frac{\Omega_l(1-\rho^{2(l+\delta-1)})}{m} \right)} e^{\frac{-mz_l^2}{\Omega_l(1-\rho^{2(l+\delta-1)})}} \times e^{\frac{-\rho^{2(l+\delta-1)}t}{1-\rho^{2(l+\delta-1)}}} \times {}_0F_1 \left(; m; \frac{mz_l^2 \rho^{2(l+\delta-1)}t}{\Omega_l(1-\rho^{2(l+\delta-1)})^2} \right) dt, \quad (4)$$

where $\Gamma(\cdot)$ is the complete gamma function [14, Eq. 6.1.1], ${}_0F_1(\cdot; \cdot; \cdot)$ the confluent hypergeometric limit function [15, Eq. 9.14.1], m denotes the Nakagami- m fading order, ρ is the time correlation, δ is the channel feedback delay and $\Omega_l = \mathbb{E}\{|h_{ca}(l)|^2\}$.

Denoting the mutual information for each consecutive transmission k , $1 \leq k \leq L$, as

$$I_{ca}(k) = B \log_2(1 + \gamma_{ca}(k)), \quad (5)$$

the outage probability of L consecutive transmissions is

$$\mathcal{O}_L = \Pr\{I_{ca}(1) < \mathcal{R}, \dots, I_{ca}(L) < \mathcal{R}\}, \quad (6)$$

where $\mathcal{R} = B \mathcal{R}_b$ is the transmit rate and \mathcal{R}_b is the spectral efficiency.

Following [13], we can rewrite (6) in terms of the joint cumulative distribution function (CDF) of the of L independent Nakagami- m random variables, $F_{|\mathbf{h}_L|}(\cdot)$, so that

$$\mathcal{O}_L = F_{|\mathbf{h}_L|}(I_{ca}(1) < \mathcal{R}, \dots, I_{ca}(L) < \mathcal{R}) = F_{|\mathbf{h}_L|}(|h_{ca}(1)| < \varpi, \dots, |h_{ca}(L)| < \varpi), \quad (7)$$

where $\varpi = \sqrt{\frac{2^{\mathcal{R}_b} - 1}{\gamma_{ca}}}$, simplifying the notation since $\bar{\gamma}_{ca}$ is the same regardless of the channel realization,

$$F_{|\mathbf{h}_L|}(\varpi) = \sum_{n_1, \dots, n_L=0}^{\infty} \frac{\Gamma(m + \sum_{l=1}^L n_l)}{\Gamma(m) \left(1 + \sum_{l=1}^L \omega_l\right)^m} \times \prod_{l=1}^L \frac{1}{n_l!} \left(\frac{\omega_l}{1 + \sum_{l=1}^L \omega_l} \right)^{n_l} \times \prod_{l=1}^L \frac{\Upsilon\left(m + n_l, \frac{m\varpi}{\Omega_l(1-\rho^{2(l+\delta-1)})}\right)}{\Gamma(m + n_l)}, \quad (8)$$

in which $\omega_l = \frac{\rho^{2(l+\delta-1)}}{1-\rho^{2(l+\delta-1)}}$ and $\Upsilon(\cdot, \cdot)$ the lower incomplete Gamma function [14, Eq. 6.1.1].

However, the outage probability in (8) is difficult to manipulate, due to the multiple product and summation operations. Instead, it is possible to use a high SNR, which is a common assumption in URLLC systems due to the high required reliability. Under high SNR regime it is possible to obtain an asymptotic for the outage probability as [13]

$$\tilde{\mathcal{O}}_L = \frac{m^{mL} (2^{\mathcal{R}_b} - 1)^{mL} \vartheta(L, \rho)}{[\Gamma(m+1)]^L \prod_{l=1}^L \Omega_l^m P_l^m} \quad (9)$$

being

$$\vartheta(L, \rho) = \left[\left(1 + \sum_{l=1}^L \frac{\rho^{2(l+\delta-1)}}{1-\rho^{2(l+\delta-1)}} \right) \prod_{l=1}^L 1 - \rho^{2(l+\delta-1)} \right]^{-m}. \quad (10)$$

IV. OPTIMIZATION PROBLEM

In this section our goal is to minimize the energy consumption per transmitted bit (E_b) of the proposed WNCS with time-correlated Nakagami- m fading channels. The energy consumed per bit can be written as

$$E_{ca} = \frac{P_{ca}\eta^{-1} + P_{TX} + P_{RX}}{\mathcal{R}}, \quad (11)$$

where η is the efficiency of the power amplifier, P_{TX} is the power consumption of the radio frequency (RF) circuits at the transmitter and P_{RX} is the equivalent at the receiver.

Therefore, aiming at minimizing E_b we allocate the transmit power of the controller. The proposed optimization problem is written as

$$\min_{P_{ca}} E_{ca} \quad (12a)$$

$$\tilde{\mathcal{O}}_L \leq \mathcal{P}_o, \quad (12b)$$

$$0 \leq P_{ca} \leq P_{\max}, \quad (12c)$$

where P_{\max} is a maximum transmit power constraint and \mathcal{P}_o the outage probability threshold.

In order to solve (12a), let us first relax the condition in (12c). Moreover, we also know from [13] that the transmit power increases when $\tilde{\mathcal{O}}_L$ decreases, so that to minimize the transmit power we must operate at $\tilde{\mathcal{O}}_L = \mathcal{P}_o$. Then, after some algebra, the optimal transmit power under high SNR regime can be written as

$$\hat{P}_{ca} = \frac{m(2^{\mathcal{R}_b} - 1)(N_0 B)}{(\Theta \Psi)^{-L} \mathcal{P}_o [\Gamma(m+1)]^{-m}}, \quad (13)$$

where

$$\Theta = 1 + \sum_{l=1}^L \frac{\rho^{2(l+\delta-1)}}{1-\rho^{2(l+\delta-1)}}, \quad (14)$$

$$\Psi = \prod_{l=1}^L 1 - \rho^{2(l+\delta-1)}. \quad (15)$$

In summary, in order to obtain the transmission power for the minimization problem in (12a) we first determine $\tilde{\mathcal{O}}_L$ using (9), which depends on the joint CDF of the time-correlated Nakagami- m fading realizations. Then, we determine \hat{P}_{ca} using (13) and, finally, we reintroduce the constraint in (12c) by doing

$$P_{ca}^* = \max\{0, \min\{\hat{P}_{ca}, P_{\max}\}\}. \quad (16)$$

V. NUMERICAL RESULTS

In this section we present some numerical results in order to illustrate our theoretical analysis. The parameters used for modeling the communication system are summarized in Table I, following [16], while the system reliability is linked to the URLLC requirements in [17], so that we assume $\mathcal{P}_o = 10^{-5}$ for the WNCS under study.

TABLE I
COMMUNICATION SYSTEM PARAMETERS

Parameter	Value
Channel feedback delay	$\delta = 1$
Carrier frequency	$f = 2.5$ GHz
Antenna gains	$G = 5$ dB
Noise psd	$N_0 = -204$ dBW/Hz
Bandwidth	$B = 10$ kHz
Spectral efficiency	$\mathcal{R}_b = 1$ bps/Hz
Link margin	$M = 20$ dB
Path loss	$\alpha = 2.5$
Power amplifier efficiency	$\eta = 0.35$
Circuit power consumption at the TX	$P_{TX} = 97.9$ mW
Circuit power consumption at the RX	$P_{RX} = 112$ mW

Figure 2 shows the relationship between the energy consumed per bit (E_b) and the time correlation (ρ) for different Nakagami- m parameters. We optimize the transmit power, assuming $d_{ca} = 50$ m and two consecutive transmission with fading, $L = 2$. Moreover, both sensor and controller use the optimal transmit power given by (16). So, as we can observe, the energy consumption increases with ρ , which occurs since higher ρ implies in exponentially higher power consumption. In addition, Figure 2 shows that the minimal energy consumed per bit decreases with the increase of m . As expected, higher fading order yields to a higher diversity order, thus reducing the energy consumed per bit.

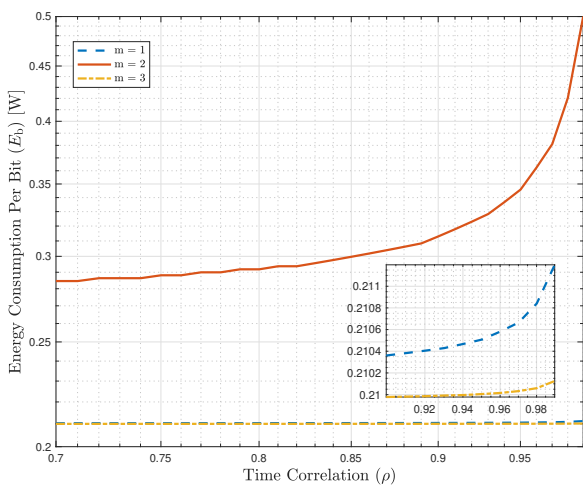


Fig. 2. Energy consumed per bit as a function of time correlation for different fading orders.

Furthermore, Figure 3 plots the energy consumed per bit

as a function of different distances between the controller and the actuator. Moreover, we also consider two consecutive transmission with fading, $L = 2$. As we can observe from the figure, the presence of time-correlation has an impact on the amount of energy consumed per bit. In addition, in this figure we have three schemes without time-correlation ($\rho = 0$) and one scheme with full time-correlation ($\rho = 1$), from which we notice that the system requires 4 times more energy compared to the case with $\rho = 0$ when the link distance is $d_{ca} = 500$ m.

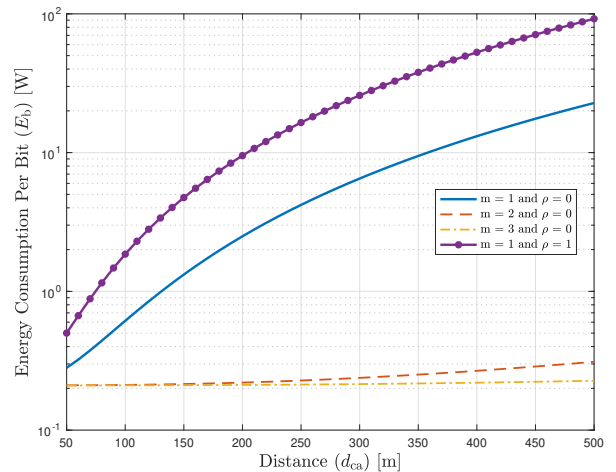


Fig. 3. Energy consumed per bit as a function of the distance between receiver and transceiver for different fading orders and time correlation.

Finally, Table II compares the relationship between energy consumption per bit and the number of consecutive outage events (L) for different distances in the wireless links. For the simulation we have used $m = 1$ and $\rho = 1$. As we can observe in Table II for one transmission with fading, $L = 1$ communicating at $d_{ca} = 400$ m requires 75% more energy when compared to $d_{ca} = 100$ m. In addition, at the distance of $d_{ca} = 50$ m, considering four consecutive transmission with fading, $L = 4$, requires $145 \cdot 10^3$ times more energy compared to the case of $L = 1$. Such increase in energy consumption is up to $18 \cdot 10^6$ times when the distance is $d_{ca} = 500$ m.

TABLE II
ENERGY CONSUMED PER BIT AS A FUNCTION OF THE NUMBER OF CONSECUTIVE OUTAGES FOR DIFFERENT DISTANCES IN THE WIRELESS LINK.

Distance	Consecutive Outage Events (L)			
	1	2	3	4
50 m	210.8 mW	500.6 mW	92.13 W	29.07 kW
100 m	215.1 mW	1.854 W	0.52 kW	0.1644 MW
200 m	239.3 mW	9.512 W	2.942 kW	0.93 MW
300 m	291 mW	0.025 kW	8.106 kW	2.563 MW
400 m	376.3 mW	0.052 kW	16.64 kW	5.262 MW
500 m	500.6 mW	0.092 kW	29.07 kW	9.192 MW

VI. CONCLUSION

In this paper we proposed a solution to minimize the energy consumption per transmitted bit of a WNCS. The goal was

to obtain the minimal required transmitted power when the WNCS is subjected to time correlation and consecutive outage events. By modeling the wireless link as a time-correlated Nakagami- m fading channel, we defined and optimization problem to minimize the energy consumption. The solution of this optimization problem yielded to a close-form expression for the optimal transmit power in this scenario. Our results show that the optimal parameters depend on the distance between the transmitter and receiver, as well as on the time correlation. More importantly, the increase in the number of consecutive outage events increases the energy consumption dramatically.

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