

CONVERSORES ANALÓGICOS PARA INFORMAÇÕES: ARQUITETURA, APLICAÇÕES E DESAFIOS

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I AM...

- Edmar Candeia Gurjão
- Professor of Electrical Engineering at Federal University of Campina Grande
- Interests:
 - Compressed Sensing
 - Software Defined Radio
 - Radiometry (RFID, Sensors, Biomedical Applications)
 - Modeling and Simulation
- **Cooperation with Prof. Raimundo Freire**



SUMMARY

- ① Introduction to Compressed Sensing
- ② Analog-to-Information Converters (AIC)
- ③ Developments at UFCG
- ④ Challenges

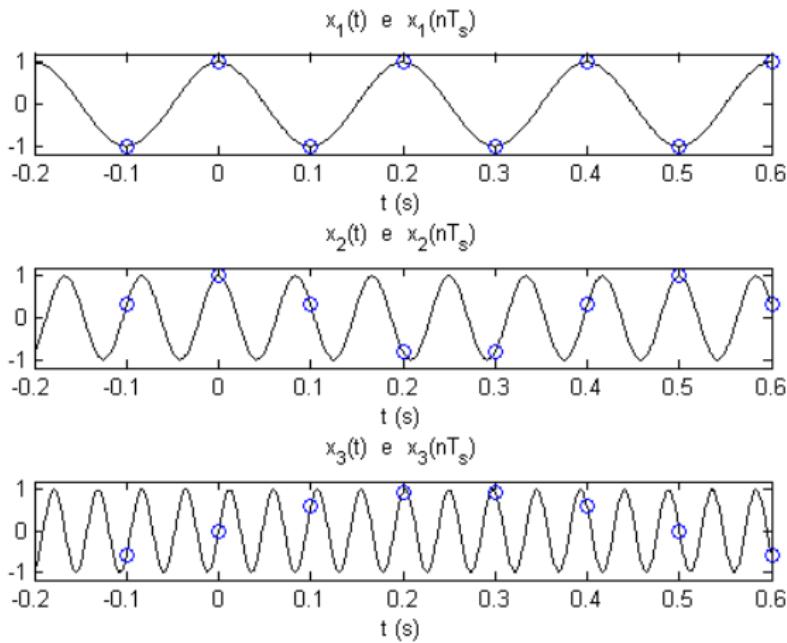


INTRODUCTION

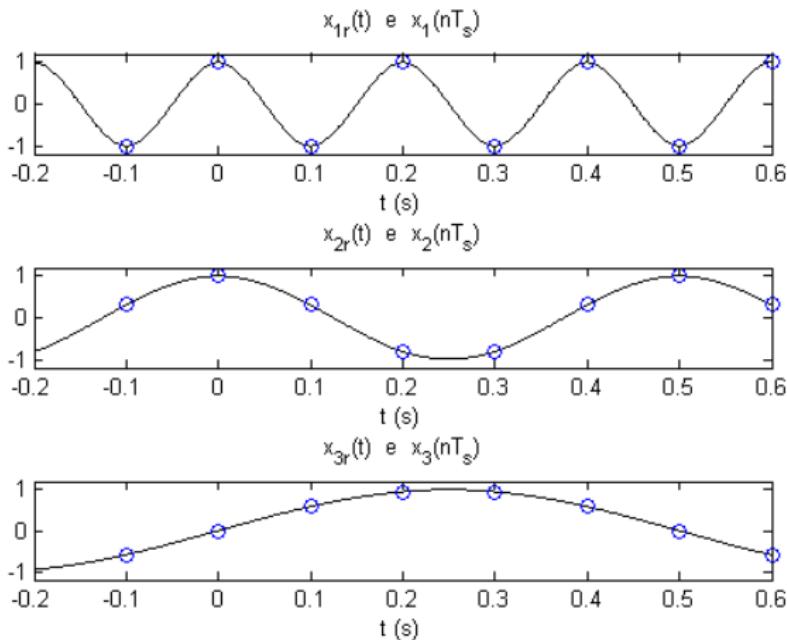
Big data:

- The amount of data generated rows a rate of 58% by year
- In 2010 it was generated 1250 billions of Gigabytes of data
 - More than stars in universe.
- Storage grows at 40% of year.
- Depending in the resolution and recording standard, obtained images of a camera have pixels discarded.
- Some audio standards discharge certain frequencies.
- **Donoho, Candès and Tao:** Why not acquire just the information of a signal?

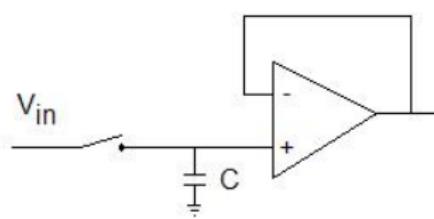
ANALOG-TO-DIGITAL CONVERTER - SAMPLING



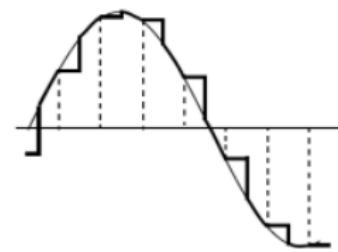
ANALOG-TO-DIGITAL CONVERTER - RECOVERING



SAMPLE AND HOLD



(a)



(b)



ANALOG-TO-DIGITAL CONVERTER

Nyquist Theorem: a limited frequency signal $x(t)$, $|X(f)| = 0, |f| > F_M$, is unequivocally determined by its samples $x(nT_S)$, $n = 0, \pm 1, \pm 2, \dots$ since $F_s = \frac{2}{T_s} \geq 2F_M$.

- Only consider the spectral content of the signal, not the information;
- Applied to any class of signals.

ANALOG-TO-DIGITAL CONVERTER - UNDERSAMPLING

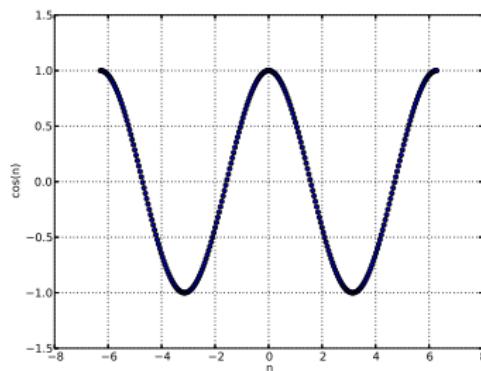
Alternative to reduce the amount of data is to undersample:



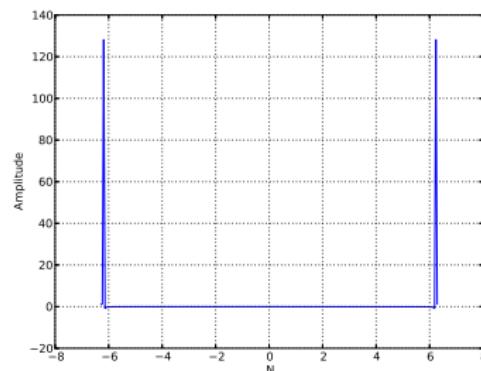
- Information can be lost.

SPARSE SIGNAL

- **Definition** A **sparse** representation of a signal with N components is sparse in certain domain if it has just $S \ll N$ non zero amplitude values.
 - Ex. sine wave: Time \times Frequency



(c)

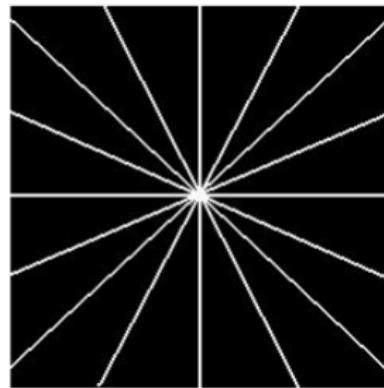


(d)



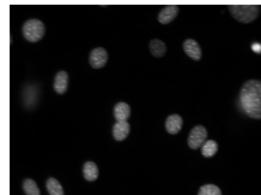
INTRODUCTION - SPARSE SIGNAL

Magnetic Resonance and its Fourier Transform

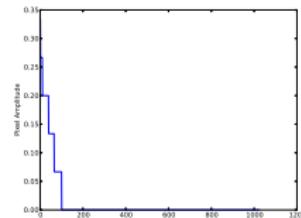


COMPRESSIBLE SIGNAL

- A **compressible** representation that approximate a signal with S non zero coefficients.



(e)



(f)

- Taking only the non-zero values and it positions we can obtain a high fidelity representation.
- Fundamentals of encoding by transforms: JPEG, JPEG2000, MPEG, MP3, etc.



COMPRESSED SENSING

- or Compressed Sampling, Compressive Sensing was raised as a framework to obtain more compact representations of sparse or compressible signals than the ones produced by converters based on Nyquist Rate.
- The basic idea is use projections in lower dimension spaces, and when recovery is necessary to use optimization.
- Not a new idea, in other contexts have been applied since 1975.



COMPRESSED SENSING

- Sparse Signal $\mathbf{x} \in \mathbb{R}^N$
 - \mathbf{x} is S -sparse: $\{i : x_i \neq 0\}$ have size equal to or lower than S
- Set of measurements (projections) \mathbf{y} given by

$$\mathbf{y}_m = \langle \mathbf{x}, \mathbf{a}_m \rangle, \quad m = 1, \dots, M.$$

- \mathbf{a}_m vectors used by measurements
- Matrix notation $\mathbf{y} = \mathbf{A}\mathbf{x}$.

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_M \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1N} \\ a_{21} & a_{22} & \cdots & a_{2N} \\ \vdots & \vdots & \vdots & \vdots \\ a_{M1} & a_{M2} & \cdots & a_{MN} \end{bmatrix} \times \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{bmatrix}$$



LINEAR SYSTEM OF EQUATIONS ($M = N$)

$$\begin{matrix} \mathbf{y} & = & \mathbf{A} & \times & \mathbf{x} \\ \begin{matrix} y_1 \\ y_2 \\ \vdots \\ y_M \end{matrix} & = & \begin{matrix} a_{1,1} & & & \dots & a_{1,N} \\ a_{2,1} & & & \dots & a_{2,N} \\ \vdots & & & & \vdots \\ a_{M,1} & & & \dots & a_{M,N} \end{matrix} & \times & \begin{matrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{matrix} \end{matrix}$$



LINEAR SYSTEM OF EQUATIONS ($M > N$)

$$\begin{matrix} \mathbf{y} \\ y_1 \\ y_2 \\ \vdots \\ y_M \end{matrix} = \begin{matrix} \mathbf{A} \\ a_{1,1} & & & \dots & a_{1,N} \\ a_{2,1} & & & \dots & a_{2,N} \\ \vdots & & & & \vdots \\ \vdots & & & \dots & \vdots \\ a_{M,1} & & & & a_{M,N} \end{matrix} \times \begin{matrix} \mathbf{x} \\ x_1 \\ x_2 \\ \vdots \\ x_M \end{matrix}$$



LINEAR SYSTEM OF EQUATIONS ($M < N$)

$$\begin{matrix} \mathbf{y} \\ y_1 \\ y_2 \\ \vdots \\ y_M \end{matrix} = \begin{matrix} \mathbf{A} \\ a_{1,1} & \cdots & a_{1,N} \\ a_{2,1} & \cdots & a_{2,N} \\ \vdots & & \vdots \\ a_{M,1} & \cdots & a_{M,N} \end{matrix} \times \begin{matrix} \mathbf{x} \\ x_1 \\ x_2 \\ x_3 \\ x_4 \\ \vdots \\ x_N \end{matrix}$$



LINEAR SYSTEMS × SIGNAL REPRESENTATION

Back to linear systems:

$$\begin{bmatrix} y_1 \\ \vdots \\ y_M \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1N} \\ \vdots & \vdots & \vdots & \vdots \\ a_{M1} & a_{M2} & \cdots & a_{MN} \end{bmatrix} \times \begin{bmatrix} x_1 \\ \vdots \\ x_N \end{bmatrix}$$

After multiplication we obtain for the first line:

$$y_1 = a_{11}x_1 + a_{12}x_2 + \dots + a_{1N}x_N$$

y_1 has information about all x_i , weighted by the respective a_{1i} .

- Knowing the N weightings, we can determine x_i .
- Doing $N > M$ we have error correction.

In these cases we can recover the original signal by $\mathbf{x} = [\mathbf{A}^T \mathbf{A}]^{-1} \mathbf{A}^T \mathbf{y}$.



COMPRESSED SENSING

- Our interest is $M < N$
- Less measurements than values in the signal (compression)
- $\mathbf{y} = \mathbf{Ax}$ has more unknowns than equations
 - no solution or infinite number of solutions.
 - Lets consider matrix \mathbf{A} is full rank, i.e their columns (*span*) all \mathbb{R}^N ,
- If we know (oracle) the null positions of \mathbf{x} , the set \mathcal{S} , we can form the matrix $\mathbf{A}_{\mathcal{S}}$ only with the columns indicating by \mathcal{S} and to solve
$$\mathbf{x}_{\mathcal{S}} = [\mathbf{A}_{\mathcal{S}}^T \mathbf{A}_{\mathcal{S}}]^{-1} \mathbf{A}_{\mathcal{S}}^T \mathbf{y}.$$
- Without the previous knowledge and \mathbf{x} sparse: optimization method.
 - Oracle solution serve as a reference.

COMPRESSED SENSING

- l_p norm of a vector $(\| \mathbf{x} \|_p)^p = \sum_{i=1}^N |x_i|^p$
- l_0 norm counts the number of non-zero elements in a vector, i.e. its support.
- Signal \mathbf{x} can be recovered from measurements \mathbf{y} by solving the optimization

$$(P_0) \min_{\tilde{\mathbf{x}} \in \mathbb{R}^N} \| \tilde{\mathbf{x}} \|_{l_0} \text{ subject to } \mathbf{A}\tilde{\mathbf{x}} = \mathbf{y},$$

- Solution of this problem involves search in $\binom{N}{S}$ possible supports.



COMPRESSED SENSING

- Search in $\binom{N}{S}$ possible supports.
- As the sparsity increases (lower values of S) the number of possibilities increases.
- Example: [Elad's book] For $M = 500$ and $N = 2000$, if a signal has sparsity $S = 20$ we have

$$\binom{N}{S} \approx 3,9 \times 10^{47} \text{ possibilities}$$

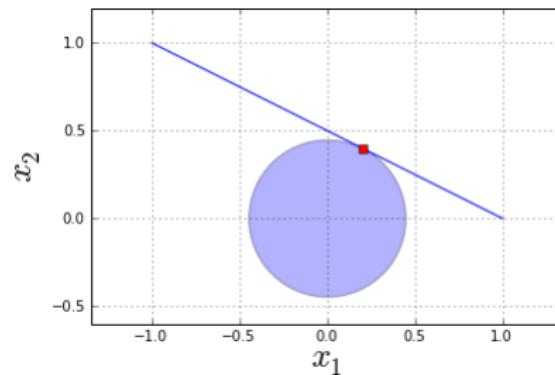
- NP-complete problem.

COMPRESSED SENSING

- Alternative to the P_0 :

- l_2 norm: $(\| \mathbf{x} \|_2)^2 = \sum_{i=1}^N |\mathbf{x}_i|^2$

- Ex.: $\mathbf{x}^T = [1, 0]$, $A = [1]$ then $\mathbf{y} = [1]$;
- optimization consists in to search in the ball:



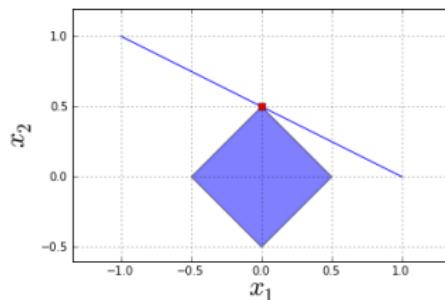
- $\hat{\mathbf{x}}^T = [\frac{1}{\sqrt{2}} \quad \frac{1}{\sqrt{2}}]$, is a solution, but not sparse.



COMPRESSED SENSING

- Alternative to the P_0 :

- l_1 norm: $(\| \mathbf{x} \|_1)^1 = \sum_{i=1}^N |\mathbf{x}_i|$
- Ex.: $\mathbf{x}^T = [1, 0]$, $A = [1]$ then $\mathbf{y} = [1]$;
- optimization consists in t search in the ball:



- $\hat{\mathbf{x}}^T = [1 \ 0]$, is a sparse solution.
- Taking $M \geq S \log_2(N/S) \ll N$ a signal can be recovered solving

$$(P_1) \min_{\tilde{\mathbf{x}} \in \mathbb{R}^n} \| \tilde{\mathbf{x}} \|_1 \text{ subject to } \mathbf{A}\tilde{\mathbf{x}} = \mathbf{y},$$



OTHER FORM TO WRITE THE OPTIMIZATION PROBLEM

$$\min_x \underbrace{\|Ax-y\|_2^2}_{\text{data fidelity}} + \lambda \underbrace{\|\Phi(x)\|_1}_{\text{transform sparsity}}$$

How choose the value of M ?

- $M \geq S \log_2(N/S) \ll N$
- Minimizing recovery error: $E\|\tilde{x} - x\|^2 < \epsilon$

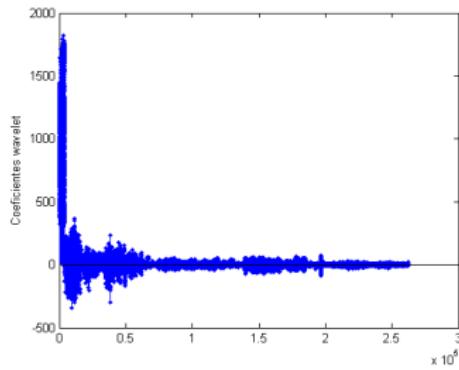


COMPRESSED SENSING

- Considering \mathbf{z} a non sparse signal:
- to apply a transform Φ and to obtain a sparse representation $\mathbf{x} = \Phi\mathbf{z}$
- Φ ortonormal, $\phi\Phi^H = \phi^H\Phi = \mathbf{I}$, where Φ^H is the hermitian transpose



(g) Lena



(h) DWT Coefficients

COMPRESSED SENSING

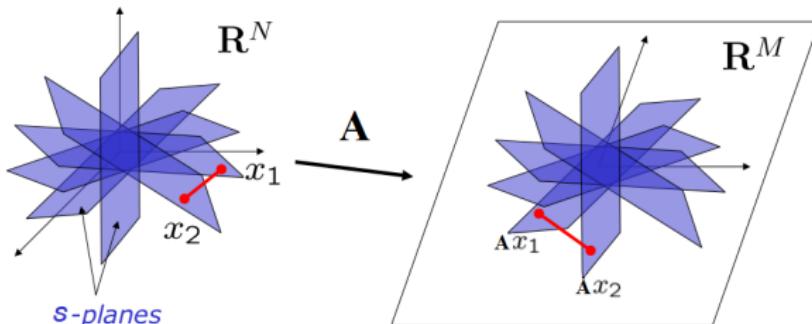
- Matrix \mathbf{A} has to permit the recovery of a S -sparse vector \mathbf{v} such that for $\delta_S > 0$

$$(1 - \delta_S) \|\mathbf{v}\|_2^2 \leq \|\mathbf{Av}\|_2^2 \leq (1 + \delta_S) \|\mathbf{v}\|_2^2.$$

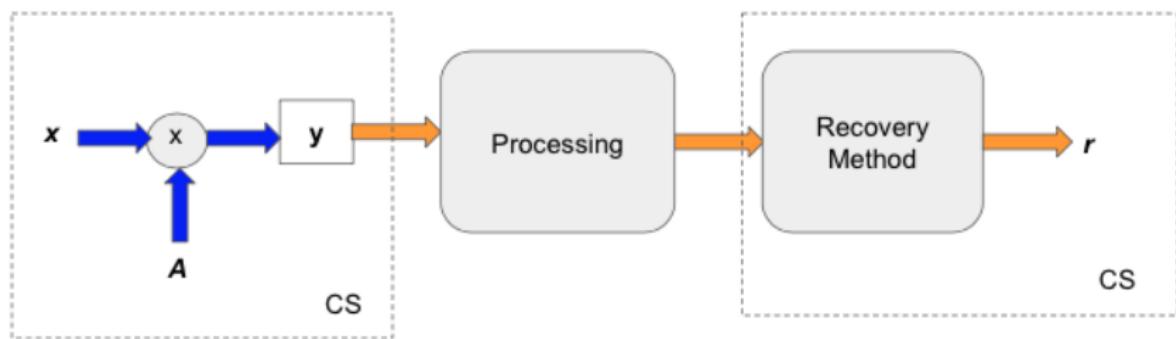
Restricted Isometry Property - RIP.

- A RIP of order $2S$ for two S -sparse, \mathbf{x}_1 e \mathbf{x}_2 :

$$(1 - \delta_{2S}) \leq \frac{\|\mathbf{Ax}_1 - \mathbf{Ax}_2\|_2^2}{\|\mathbf{x}_1 - \mathbf{x}_2\|_2^2} \leq (1 + \delta_{2S}). \quad (1)$$



Compressed Sensing Framework:



COMPRESSED SENSING - HARDWARE

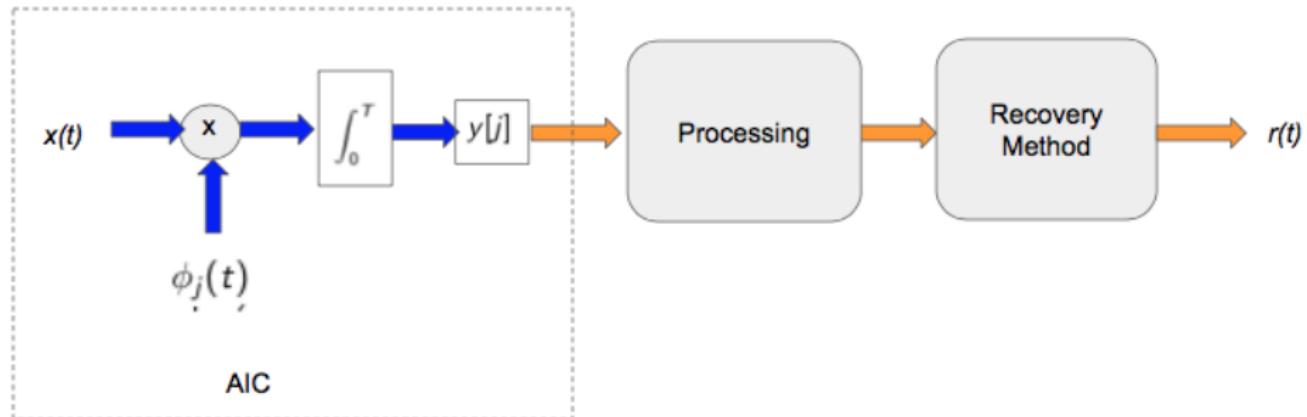
- Conversion of analog signal to digital maintaining information
- **Analog to Information Converter** - AIC.
- Signal $x(t)$, $t \in [0, T]$, with a set of test functions $\{\phi_j(t)\}_{j=1}^M$, to realize M measurements

$$y[j] = \int_0^T x(t) \phi_j(t) dt. \quad (2)$$

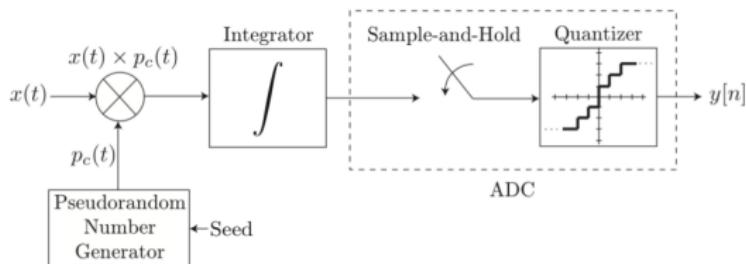
- To build such systems we need three components:
 - hardware to generate test signals $\phi_j(t)$
 - M correlators to multiply the signal $x(t)$ with each $\phi_j(t)$
 - M integrators with zero initial state.



Compressed Sensing Framework:



COMPRESSED SENSING - RANDOM MODULATOR



$x(t)$ is correlated with a sequence of random pulses ± 1 , that alternate between values in a Nyquist rate $N_a \text{Hz}$ proportional to the Nyquist rate of $x(t)$.

Mixed singal is integrated in a period of $1/M_a$ and sampled by an ADC with rate $M_a \text{Hz} \ll N_a \text{Hz}$:

$$y[j] = \int_{(j-1)/M_a}^{j/M_a} p_c(t)x(t)dt.$$



COMPRESSED SENSING

Denoted by $p_c[n]$ as the sequence of symbols ± 1 used to generate $p_c(t)$, we have $p_c(t) = p_c[n]$, $t \in [(n - 1)/N_a, 1/N_a]$, for example $j = 1$, we have

$$y[1] = \int_0^{1/M_a} p_c(t)x(t)dt = \sum_0^{N_a/M_a} p_c[n] \int_0^{1/M_a} x(t)dt \quad (4)$$

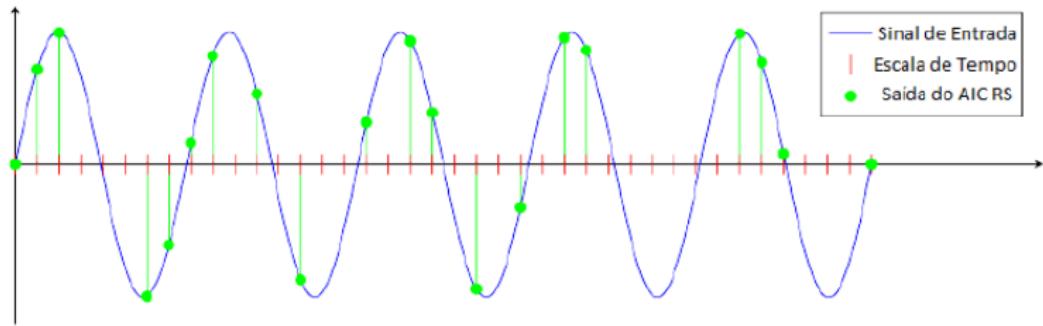
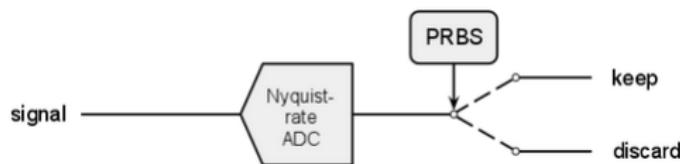
as N_a is the Nyquist rate of $x(t)$ then $\int_0^{1/M_a} x(t)dt$ is the average $x(t)$ in the n -th interval, que pode ser denotado por $x[n]$, o que nos leva a

$$y[1] = \sum_{n=1}^{N_a/M_a} p_c[n]x[n]. \quad (5)$$



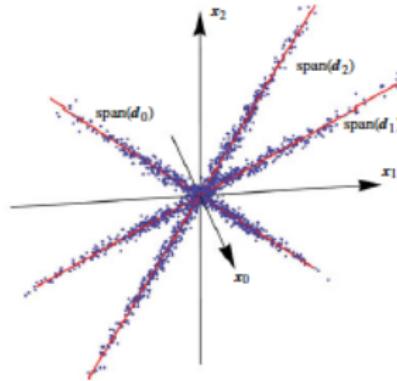
COMPRESSED SENSING - NUS

Non-uniform sampler (NUS): main part of the samples.



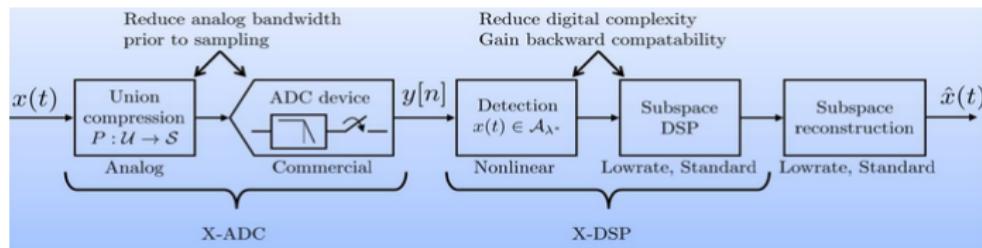
COMPRESSED SENSING - XAMPLING

Based on subspace union to determine what subspaces signal samples are concentrated.



COMPRESSED SENSING - XAMPLING

Based on subspace union to determine what subspaces signal samples are concentrated.



COMPRESSED SENSING - UFCG IMPLEMENTATION

Configurable Analog to Information Converter (Phd Thesis - UFCG)

Conversor Configurável Analógico para Informação

Vanderson de Lima Reis



COMPRESSED SENSING - UFCG IMPLEMENTATION

$$y[1] = \sum_{n=1}^{N_a/M_a} \mathbf{p}_c[n]x[n]. \quad (6)$$

Implemented in FPGA

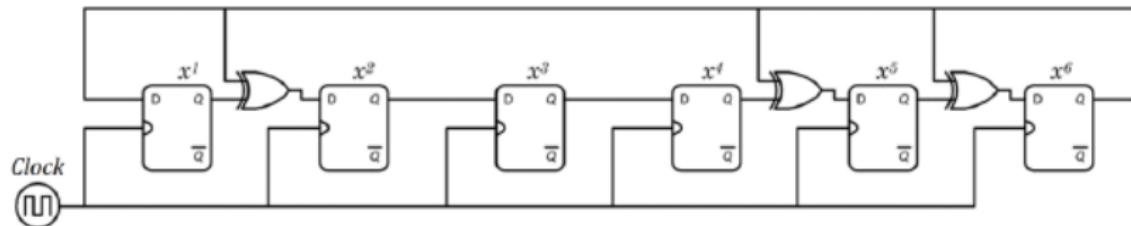


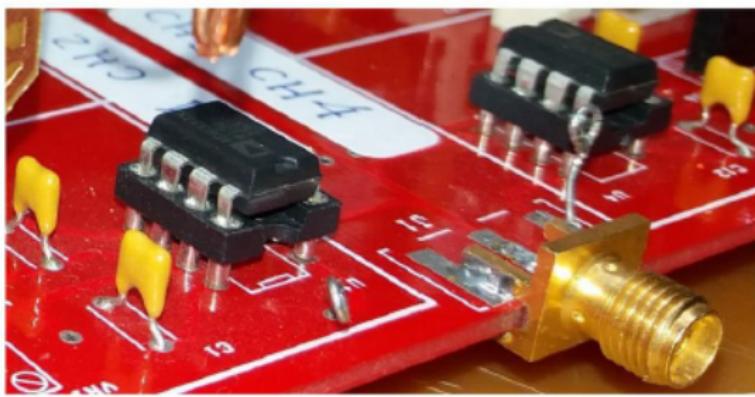
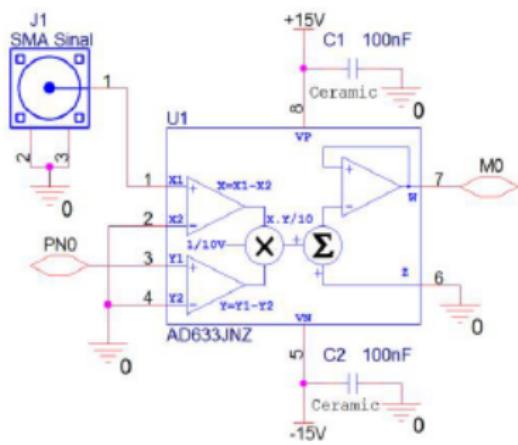
Figura 3.3: LFSR Arquitetura de Galois (ou *One-to-Many*) para o Polinômio $1 + x + x^4 + x^5 + x^6$



COMPRESSED SENSING - UFCG IMPLEMENTATION

$$y[1] = \sum_{n=1}^{N_a/M_a} p_c[n] \cdot x[n]. \quad (7)$$

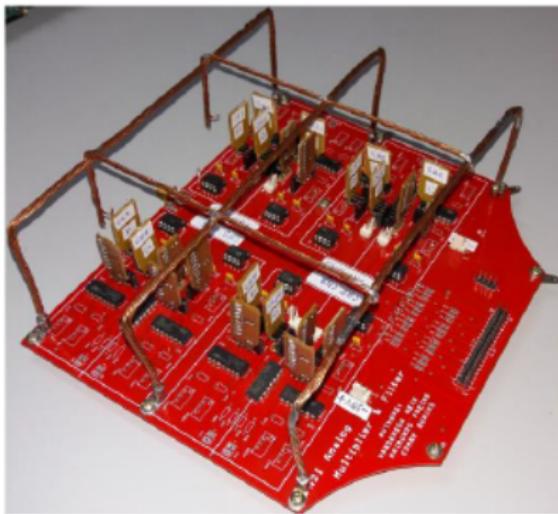
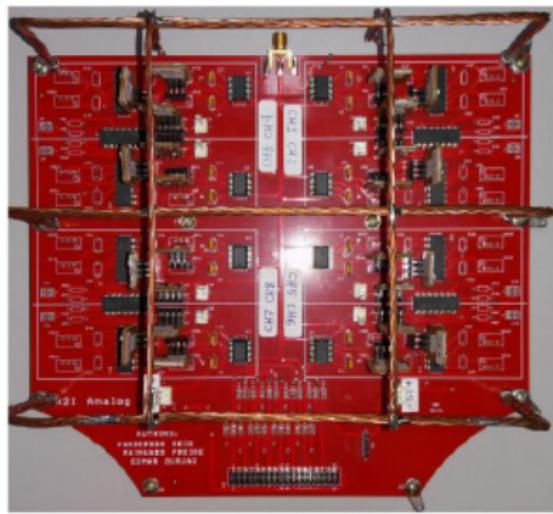
Dedicated Circuit



COMPRESSED SENSING - UFCG IMPLEMENTATION

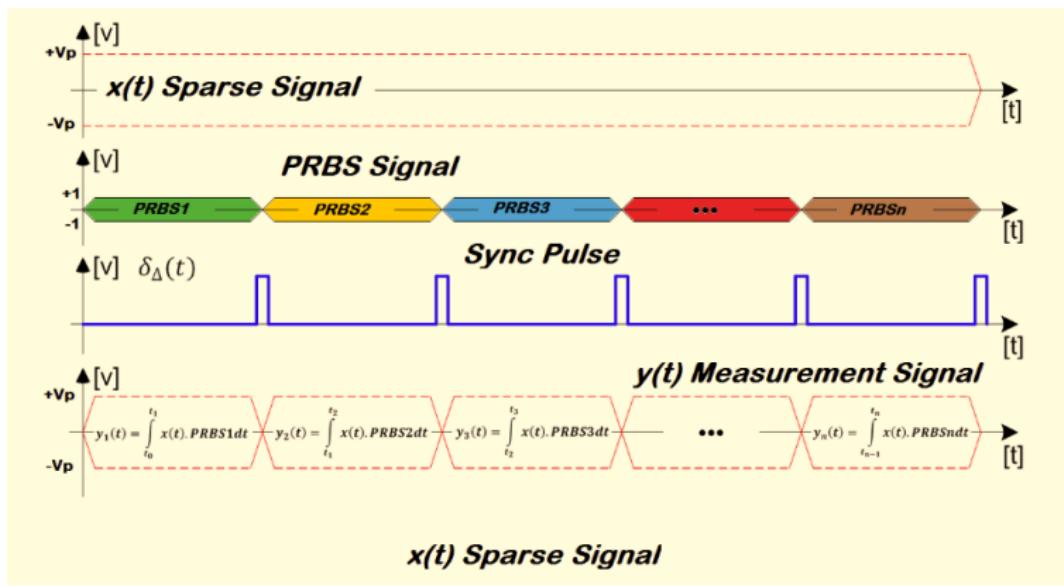
$$y[1] = \sum_{n=1}^{N_a/M_a} p_c[n] \cdot x[n]. \quad (8)$$

Dedicated Circuit

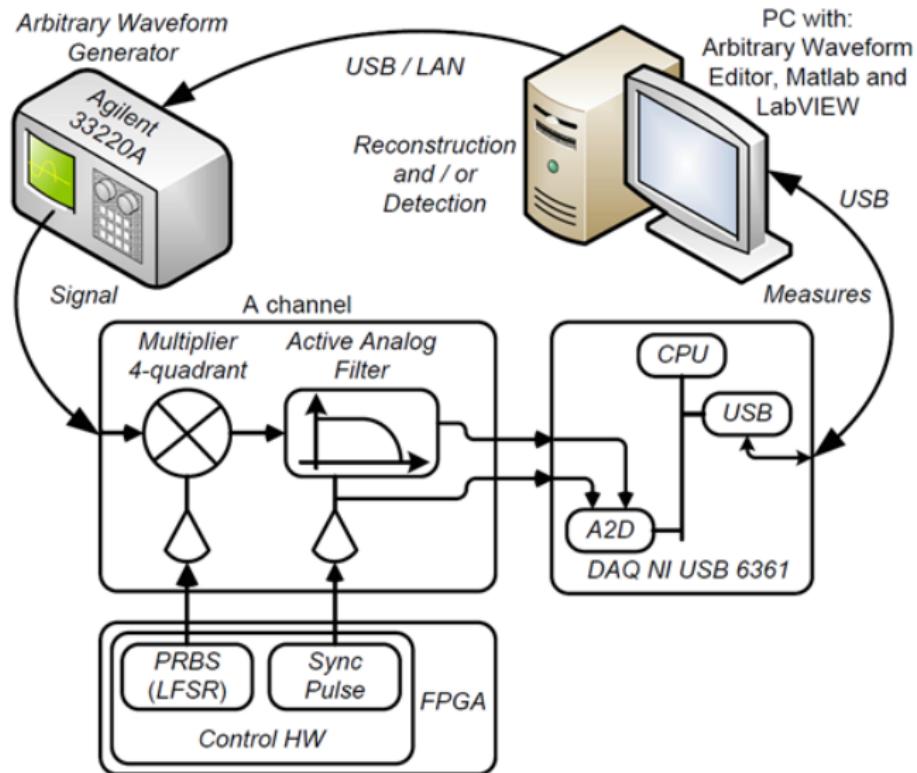


COMPRESSED SENSING - UFCG IMPLEMENTATION

$$y[1] = \sum_{n=1}^{N_a/M_a} p_c[n]x[n]. \quad (9)$$



COMPRESSED SENSING - UFCG IMPLEMENTATION



COMPRESSED SENSING - UFCG IMPLEMENTATION

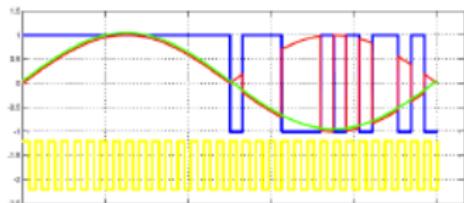


AIC

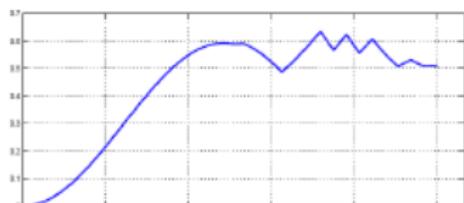


COMPRESSED SENSING - UFCG IMPLEMENTATION

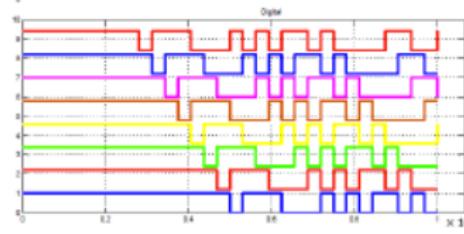
Configurable Analog to Information Converter (Phd Thesis - UFCG)



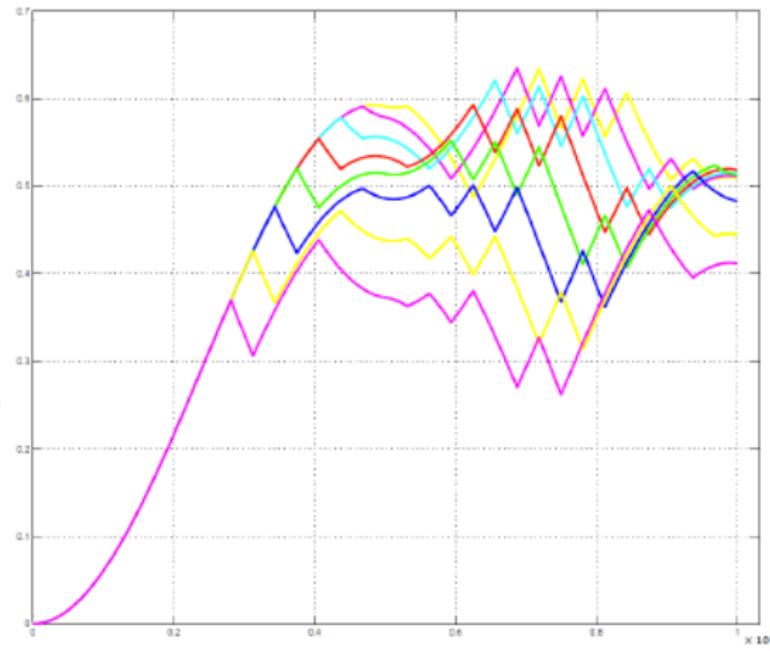
a)



b)



c)

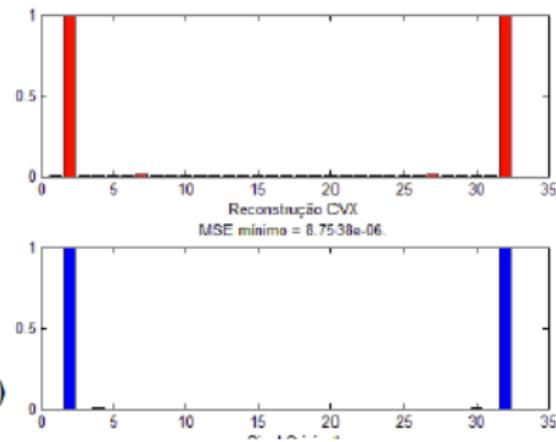
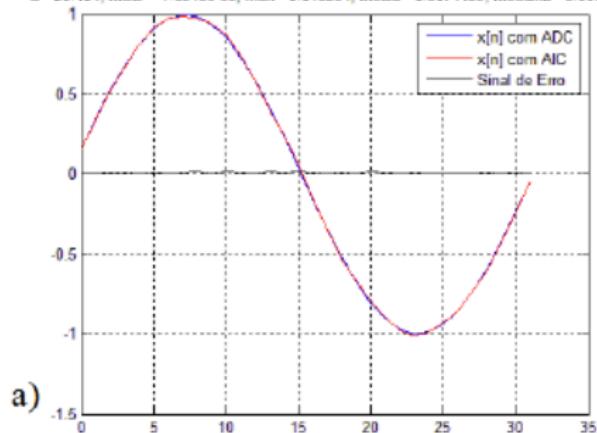


d)

COMPRESSED SENSING - UFCG IMPLEMENTATION

Configurable Analog to Information Converter (Phd Thesis - UFCG)

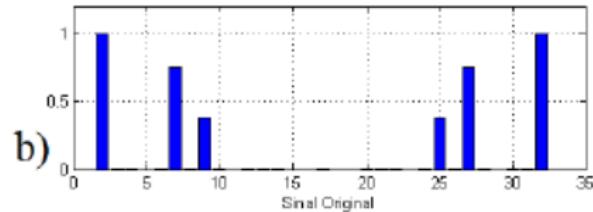
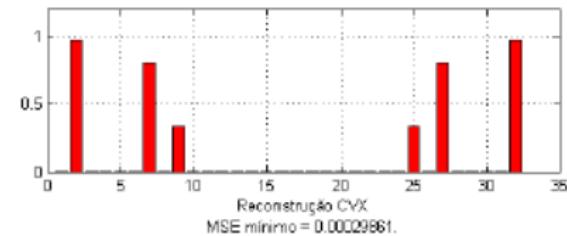
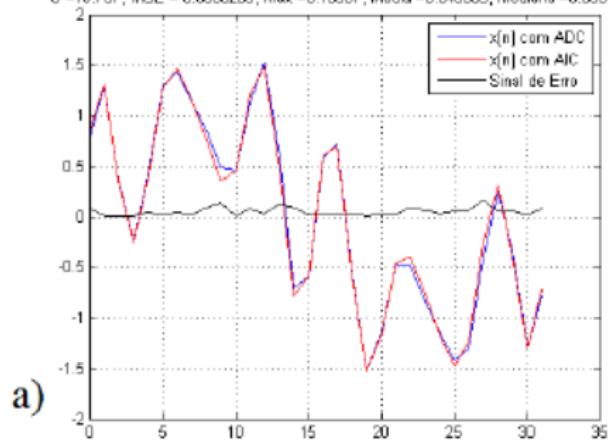
$G=20.104$, MSE = $7.8343e-05$, Max = 0.018361 , Média = 0.0074439 , Mediana = 0.0067108 .



COMPRESSED SENSING - UFCG IMPLEMENTATION

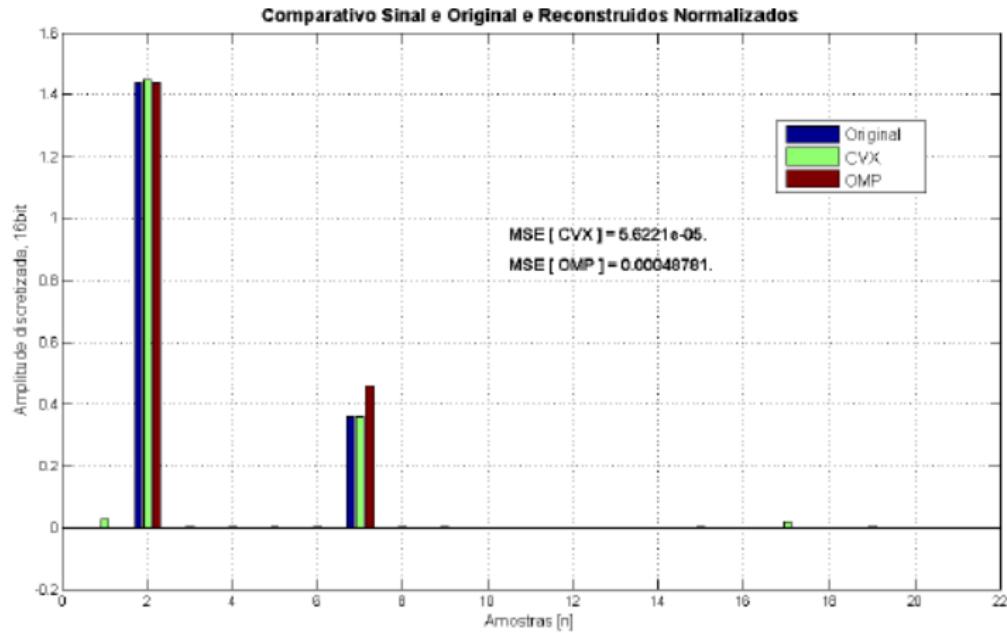
Configurable Analog to Information Converter (Phd Thesis - UFCG)

$G = 19.737$, MSE = 0.0038285, Max = 0.15307, Média = 0.048535, Mediana = 0.035159.



COMPRESSED SENSING - UFCG IMPLEMENTATION

Consider $s(t) = \sum_{i=1}^n a_i p(t - iL)$, where a_i is r.v in {0, 1}, and $p(t)$ a rectangular pulse



COMPRESSED SENSING - UFCG IMPLEMENTATION

Test Methods for Analog to Information Converters Based on IEEE 1241 Standard (Phd Thesis - UFCG)

MÉTODOS DE TESTES DE CONVERSORES ANALÓGICO
PARA INFORMAÇÃO BASEADOS NO PADRÃO IEEE 1241

Veronica Maria Lima Silva

Novel IEEE-STD-1241 based Test Methods for Analog-to-Information Converter

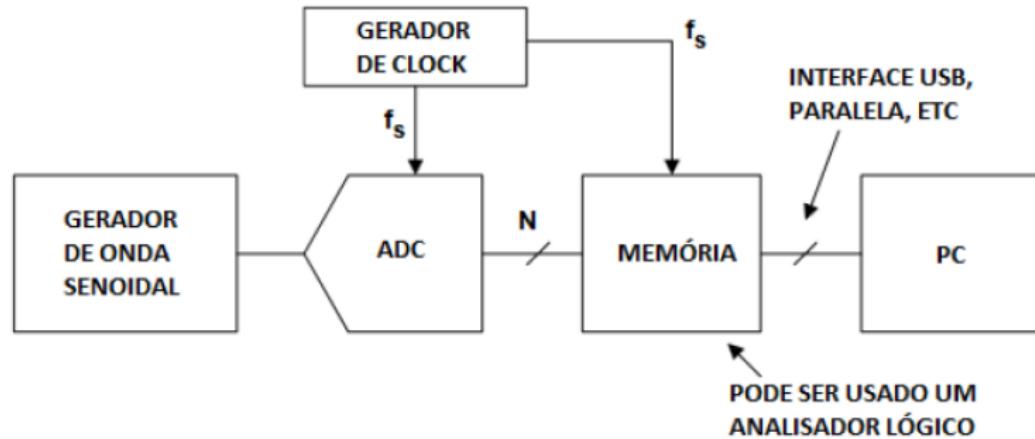
Veronica M. L. Silva ; Cleonilson P. Souza ; Raimundo C. S. Freire ; Bruno W. S. Arruda ; Edmar C. Gurjão ; Vanderson L. Reis

IEEE Transactions on Instrumentation and Measurement



COMPRESSED SENSING - UFCG IMPLEMENTATION

Sine-wave fitting for ADC



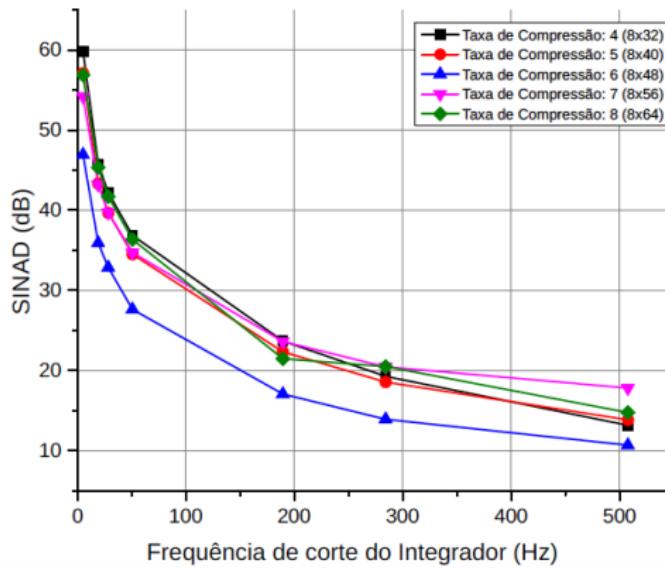
COMPRESSED SENSING - UFCG IMPLEMENTATION

Sine-wave fitting for AIC

- Input signal $x(t) = A_0 \cos(2\pi f_0 t) + B_0 \sin(2\pi f_0 t) + C_0;$
- Measurements $y;$
- Recovered sparse signal $s_0 = ((\Phi D_0)^T (\Phi D_0))^{-1} (\Phi D_0)^T y;$
- Where $s_0 = [A_0 \ B_0 \ C_0];$
- SINAD is the error between the original signal and the one recovered by the equations;

COMPRESSED SENSING - UFCG IMPLEMENTATION

Sine-wave fitting for ADC



COMPRESSED SENSING - UFCG IMPLEMENTATION

Calibration Methods Based on Models for RMPI Analog to Information
Converters (Phd Thesis - UFCG)

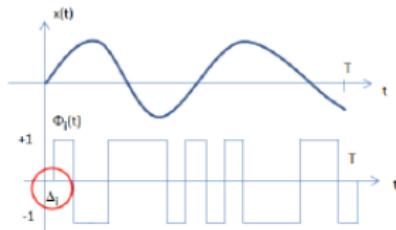
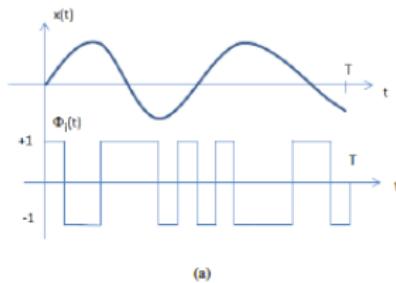
Métodos de Calibração Baseados em Modelo para Conversores
Analógicos-para-Informação do Tipo Pré-Integrador de Modulação Aleatória

Bruno Willian de Souza Arruda



COMPRESSED SENSING - UFCG IMPLEMENTATION

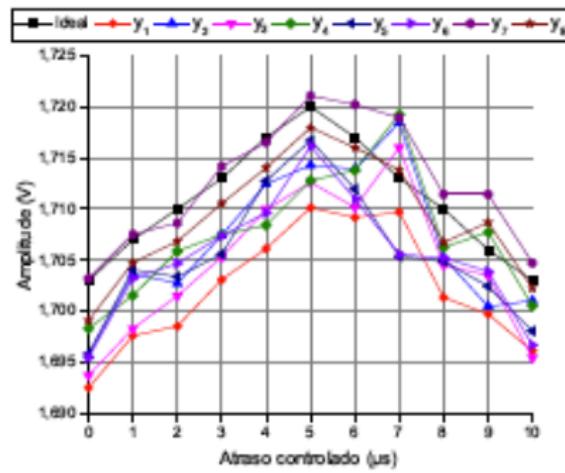
Calibration Methods Based on Models for RMPI Analog to Information Converters (Phd Thesis - UFCG)



COMPRESSED SENSING - UFCG IMPLEMENTATION

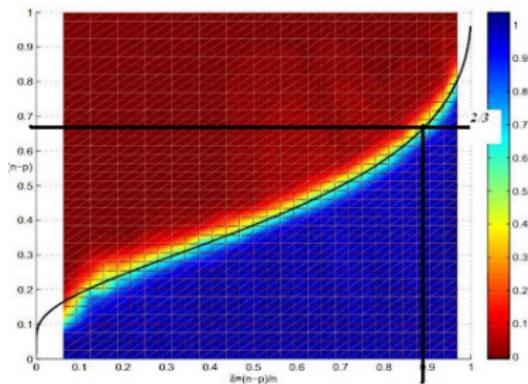
Calibration Methods Based on Models for RMPI Analog to Information Converters (Phd Thesis - UFCG)

$$\hat{y}_m = \int_0^T \Phi_m(t - \tau_m) \Phi_m(t - \Delta_m) dt.$$

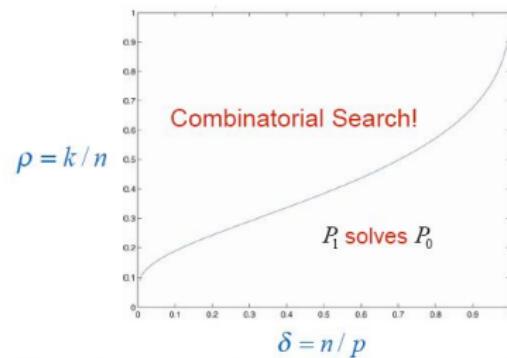


COMBINATORIAL × GREEDY ALGORITHMS

Phase transition [Donoho e Tanner]: Determine the region where a recovery method work better



(i) Observer Universality of Phase Transition. Donho e Tanner

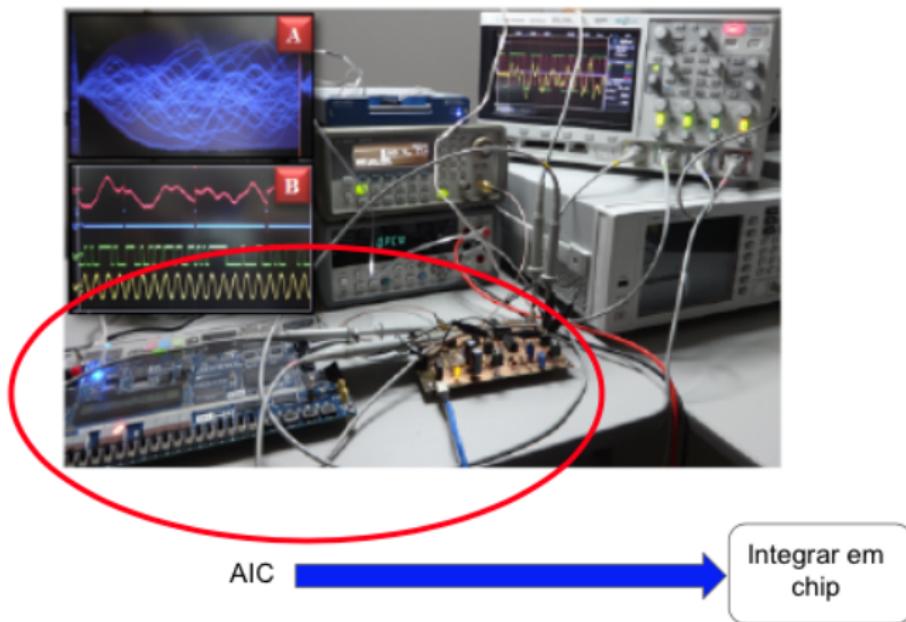


(j) Fonte: Nuit Blanche

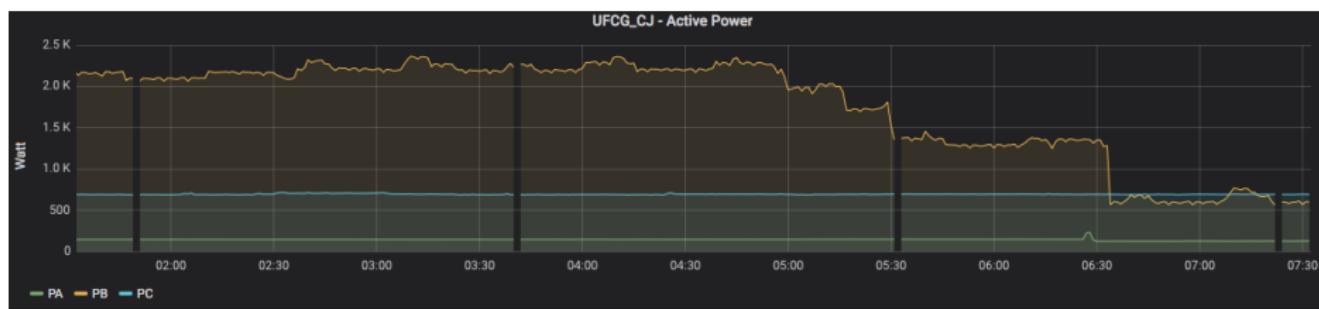
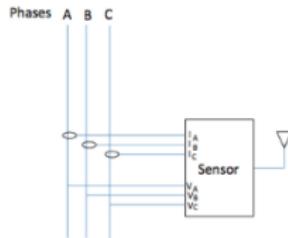
What is the Phase transition diagram of UFCG hardware?



COMPRESSED SENSING - UFCG IMPLEMENTATION



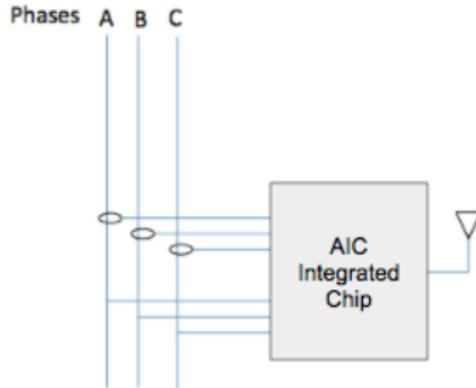
COMPRESSED SENSING - UFCG IMPLEMENTATION



(l)



COMPRESSED SENSING - UFCG IMPLEMENTATION



SIGNAL PROCESSING IN COMPRESSED DOMAIN

- Operation \mathbf{Ax} is
 - Linear
 - Restricted Isometry Property maintains signal property
- Parameters can be measured in the compressed representation
$$\mathbf{y} = \mathbf{Ax}.$$
- Some operations don't need signal reconstruction: inference, classification, estimation

IEEE JOURNAL OF SELECTED TOPICS IN SIGNAL PROCESSING, VOL. 4, NO. 2, APRIL 2010

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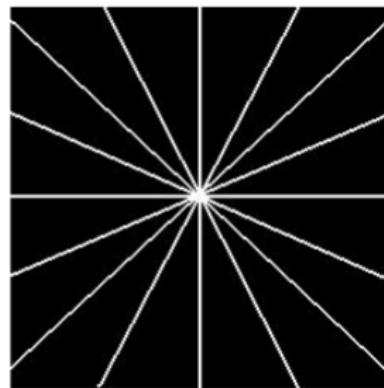
Signal Processing With Compressive Measurements

Mark A. Davenport, *Student Member, IEEE*, Petros T. Boufounos, *Member, IEEE*, Michael B. Wakin, *Member, IEEE*, and Richard G. Baraniuk, *Fellow, IEEE*



INTRODUCTION - SPARSE SIGNAL

Magnetic Resonance and its Fourier Transform

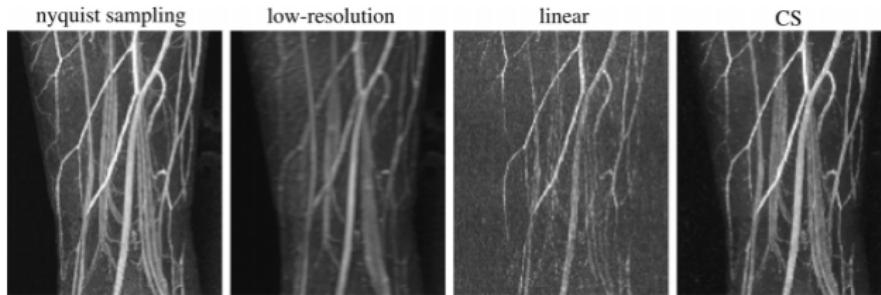


COMPRESSED SENSING – APPLICATIONS

Compressed Sensing MRI

Michael Lustig, *Student Member, IEEE*, David L. Donoho *Member, IEEE*

Juan M. Santos *Member, IEEE*, and John M. Pauly, *Member, IEEE*



See Siemens HealthCare - Compressed Sensing MRI.



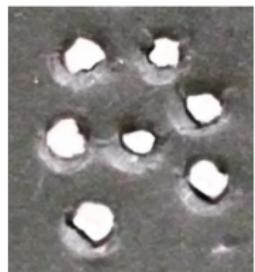


Development of Sparse Coding and Reconstruction Subsystems for Astronomical Imaging

João Maria Felner Rino Alves Silvestre



COMPRESSED SENSING – APPLICATIONS

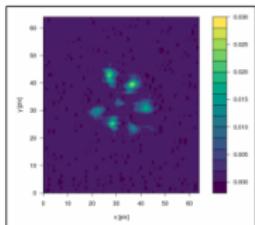


(a) 3 mm diameter punctures.

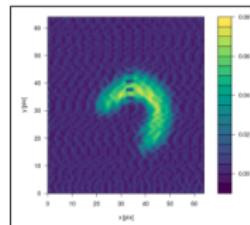


(b) A cutout 'C'.

Figure 6.13: Structures, cutout on black kapaline, that were illuminated by a white led lantern in order to test COSAC's ability to detect mixed wavelengths of visible light.



(a)



(b)

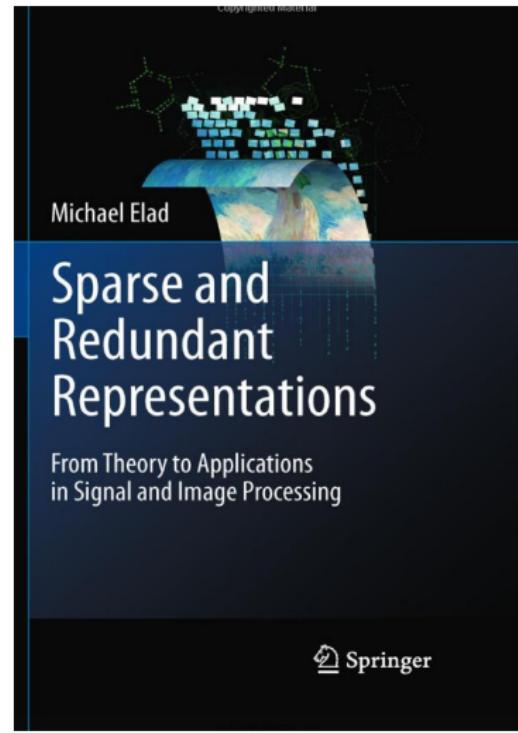
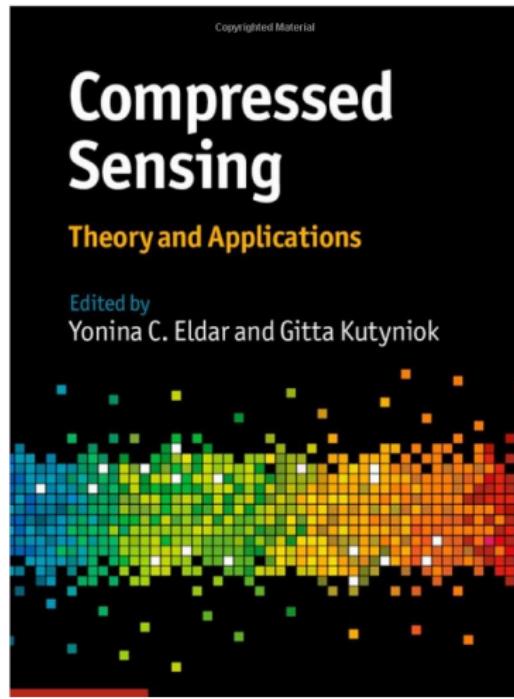


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- Nuit Blanche blog (<http://nuit-blanche.blogspot.com.br/>)
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- LUIZ CARLOS PACHECO RODRIGUES VELHO

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 **Detalhes**

CONCLUSIONS

- Compressed Sensing is in development;
- Signal Processing in the compressed domain (few works);
- Noise Folding: big challenge;
- Analog to Information Converters
 - New architectures
 - MRI is the breakthrough application
 - **Open area!**

THANK YOU!

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ecg@dee.ufcg.edu.br

<http://ecandeia.dee.ufcg.edu.br>

