XXXVII SIMPÓSIO BRASILEIRO DE TELECOMUNICAÇÕES E PROCESSAMENTO DE SINAIS - SBrT2019, 29/09/2019–02/10/2019, PETRÓPOLIS, RJ

Parity Information Aided Iterative Multistage Decoding for Multilevel LDPC Codes

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***Abstract*— Iterative demodulation and decoding (ID) seeks to extract useful information from decoders by performing multiple successive demodulation and decoding iterations. Multistage decoding with iterative demodulation and decoding (MSD-ID) unites MSD and ID, in order to improve error probability performance for multilevel codes. This article evaluates MSD-ID performance for multilevel LDPC codes and compares to a proposed MSD-ID modification called *parity information aided iterative multistage decoding* (PIA-MSD-ID), where parity-checking information obtained from the LDPC decoders is used to aid the whole decoding process. Rate distribution designs are made for MSD, MSD-ID and PIA-MSD-ID. The results show that PIA-MSD-ID achieves lower frame error rate, in comparison to multistage decoding (MSD) and similar to MSD average complexity, for higher SNR.**

***Keywords*— LDPC, Multilevel Codes, MLC, Multistage Decoding, MSD, Iterative Decoding, MSD-ID, Parity Information Aided**

I. Introduction

Multilevel coding is a coded modulation technique based on the protection of each binary digit of a constellation symbol by an individual code [1]. One convenient decoding method consists in serially decoding all codes, from the first level to the last, passing forward previous decodings' output. These are taken by the following levels as correct decisions and become a prior information to aid proceeding probability calculations. This decoding method is called *multistage decoding* (MSD).

Iterative demodulation and decoding (ID) schemes for many coded modulations have been proven to decrease error probability rates in comparison to the conventional decoding procedures [2] [3]. Wang and Burr [2] proposed the union of multilevel codes and iterative decoding. They have obtained promising results regarding error probability performance when applying multistage iterative demodulation and decoding (MSD-ID) to multilevel convolutional and turbo codes.

In this paper a MSD-ID modification is presented, named *parity information aided iterative multistage decoding* (PIA-MSD-ID). It makes use of LDPC's parity check information to detect beforehand whether there was any parity errors in one of the LDPC sum-product decoders. In order to achieve best error probability for the fixed coding rate $R = 1$, optimal rate distribution designs are made for MSD, MSD-ID and PIA-MSD-ID.

Simulation results show that both MSD-ID and PIA-MSD-ID have an improved frame error rate performance over MSD, when the optimal rate distributions are employed. PIA-MSD-ID has shown similar to MSD average complexity for higher SNR.

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II. Multilevel Codes

Let $\mathbf{A}$ be the set of all modulation symbols and $\mathcal{M}$ be a mapping function, such that $\mathcal{M}\{\mathbf{x} = (x^0, x^1, ..., x^{L-1})\} = a$, where $a \in \mathbf{A}$ is a constellation symbol, and $\mathbf{x} = (0, 1)^L$. Each bit index of $\mathbf{x}$ corresponds to a coding level. In other words, multilevel coding assigns individually a code to each bit of $\mathbf{x}$. So, if $\mathbf{x}$ consists of L bits, there will be L different codes protecting each bit separately.

A. Multistage Decoding (MSD)

Consists of decoding serially each coded level of the received symbols with its respective code, starting from the lowest level to the highest. Previous decoding levels' hard-decided outputs are taken by the higher levels as correct decisions and are used to aid bit probability calculations.

The process starts at level $\ell = 0$, where there is no previous levels' information, thus no preceding knowledge exists to be passed to decoder $\mathcal{C}^0$. The decoding decision over x^0 is made and passed ahead to level $\ell = 1$. Now, at $\ell = 1$, the value assigned to x^0 is taken as correct, the *a priori* probability of the current level's binary digit x^0 being 0 or 1 is calculated and then sent to $\mathcal{C}^1$ to be processed; x^1 is decided and passed over to the next level. This goes all the way to level $\ell = L - 1$, when the last bit x^{L-1} is decided and the MSD procedure finalized.

B. Multistage Iterative Decoding (MSD-ID)

MSD-ID [2] consists in passing soft extrinsic information among all L decoding levels, then starting over the process by sending the $\ell = L - 1$ level's soft extrinsic information to the first level $\ell = 0$, which will pass the new level $\ell = 0$ extrinsic information forward. The decoding is finalized when a desired number of iterations T is completed. A mapping $\mathcal{M}\{\mathbf{x}\} = a, a \in \mathbf{A}$ is defined, where $\mathbf{A}$ is the set of modulation symbols. The extrinsic probability of a bit $x^\ell \in \mathbf{x}$ being equal to $b \in (0, 1)$, given the received noisy symbol y is directly proportional to

$$\Pr^{demod}_{ext}\{x^\ell = b\} \sim \frac{\Pr\{x^\ell = b|y\}}{\Pr^{demod}_{prior}\{x^\ell = b\}}$$

$$= \sum_{a_t \in \mathbf{A}^\ell_b} f_Y(y|a_t) \prod_{j=0, j\neq\ell}^{L-1} \Pr\{x^j = \text{bit}(a_t, j)\} \quad (1)$$

where $\Pr^{demod}_{ext}$ is the extrinsic probability obtained from the soft demodulation, $\Pr^{demod}_{prior}$ is the *a priori* probability received from the previous decoder, $\mathbf{A}_b^\ell$ is the set of all symbols whose inverse mappings $\mathcal{M}^{-1}(a) = \mathbf{x}$ have the ℓ^{th} bit equal to b, $f_Y(y|a_t)$ is the probability density function of the received symbol being equal to y given that the transmitted symbol is a_t, and bit(a_t, ℓ) represents the ℓ^{th} bit of the binary signal vector correspondent to the constellation symbol a_t.

The extrinsic probability, to be passed to the next decoding level in order to employ (1) again, and calculated after a belief-propagation decoding, is defined as

$$\text{Pr}_{ext}^{dec}\{x^\ell = b\} = \frac{\text{Pr}_{posterior}^{dec}\{x^\ell=b\}}{\text{Pr}_{ext}^{demod}\{x^\ell=b\}} \qquad (2)$$

$\text{Pr}_{posterior}^{dec}$ is the *a posteriori* probability obtained from the decoder's output, and Pr_{ext}^{dec} is the extrinsic probability from the decoder's perspective, to be passed to the next demodulation stage.

III. Proposed Architecture

PIA-MSD-ID's novelty lies in the fact that it makes use of the syndromes $\mathbf{s}_\ell = \mathbf{c}_\ell \mathbf{H}_\ell^T$, where $\ell \in \{0, 1, ..., L-1\}$, to prevent unnecessary iterative decoding repetitions. In other words, at iteration i, if at least one of the L syndrome vectors do not return the all zero word, certainly at least one of the L decoded hard-decided codewords is not contained inside its respective code set. Therefore, if the decoding process halted at this point, at least one of the hard-decided codewords would assuredly be wrong. In this case, a new soft iterative decoding iteration takes place, with hopes that new valuable extrinsic information will be extracted. If, after this new iteration, all syndromes are equal to the all zero codeword $\mathbf{0}$, certainly, for all $\ell \in \{0, 1, ..., L-1\}$, $\mathbf{c}_\ell \subset \mathcal{C}^\ell$, where $\mathcal{C}^\ell$ is the ℓ^{th} level code; the decoded codewords are assumed to be correct and the proccess ends here, by hard-deciding over the *a posteriori* probabilities obtained from the decoders' output. A maximum number T of decoding iterations is set, so the process is interrupted if the decoders fail to converge to a valid codeword after T iterations.

IV. Simulations and Results

The simulations were made over the two level AWGN channel, using 4-PAM modulation, with codes of total rate $R = 1$ and length $n = 1000$. A frame error rate comparison among MSD, MSD-ID and PIA-MSD-ID for various SNR values and two rate distributions is presented in Fig. 1. The minimum number of frame errors is set to 800. Each belief-propagation decoding level $\mathcal{C}^\ell$, where $\ell \in \{0, 1, ..., L-1\}$, employ 50 iterations, and both MSD-ID and PIA-MSD-ID are set to have a maximum number of iterative decoding loop iterations between level 0 and level 1 of $T = 4$.

The best rate distribution satisfiyng $R = R^0 + R^1 = 1$ - where R^0 and R^1 are the first and second level parity-check matrices' rates, respectively - was sought for MSD-ID/PIA-MSD-ID at a SNR of 7.5dB, under the specified conditions, by doing Monte Carlo simulations for various rates and selecting the rate which gives the best frame error rate performance. The rate distribution that has delivered the best performance was $R^0 = 0.165, R^1 = 0.835$. The same proccess was performed for MSD, from which the rate distribution of $R^0 = 0.148, R^1 = 0.852$ was obtained. Also a complexity

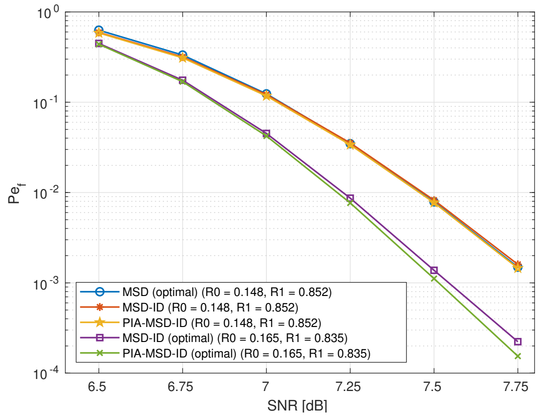

Fig. 1. Frame error rate for MSD, MSD-ID and PIA-MSD-ID

measurement of the three decoding methods is presented. The average number of belief-propagation decodings per received codeword of PIA-MSD-ID is a function of the channel SNR, so measurements were taken and are presented in Table I.

TABLE I

Complexity comparison among the decoding schemes, measured by the average belief-propagation decodings per codeword

Decoding Method	6.5dB	7dB	7.5dB	7.75dB
MSD	2	2	2	2
MSD-ID	8	8	8	8
PIA-MSD-ID	7.2819	2.8638	2.0526	2.0094

V. Conclusions

Results show that, when the optimal MSD rate distribution is set as the rate distribution for all decoding schemes, no frame error rate improvements are present. In contrast, the results also indicate that MSD-ID and PIA-MSD-ID, in comparison to MSD using the optimal MSD rate distribution, can improve frame error rate performance of multilevel LDPC codes, if an optimal rate distribution design for both schemes is performed. PIA-MSD-ID has displayed similar to MSD-ID performance with lower average complexity, accomplishing similar to MSD complexity with higher channel SNR. These results denote, therefore, that MSD-ID and PIA-MSD-ID techniques for multilevel LDPC codes are promising, if proper rate distribution designs are made.

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