

# An Efficient Grant-Based Scheme for the Massive Random Access Gaussian Channel

Gustavo Kasper Facenda and Danilo Silva

**Abstract**—This paper investigates the Massive Random Access Gaussian Channel with a focus on small payloads. For this problem, grant-based schemes have been regarded as inefficient due to the necessity of large feedbacks and use of inefficient scheduling request methods. In this paper, a novel grant-based scheme, which appears to outperform other methods, is proposed. The scheme uses Ordentlich and Polyanskiy’s grantless method to transmit small coordination indices in order to perform the scheduling request, which allows both the request from the users to be efficient and the feedback to be small. The simulation results show that, if a short feedback is allowed, our method requires lower energy per bit than existing practical grantless methods.

**Keywords**—Random Access, Massive MAC

## I. INTRODUCTION

In recent years, interest in Machine-type Communication (MTC) has increased, mostly due to the growing trend of the Internet of Things. Within MTC, a special case of interest is that when the number of devices is massively large and they transmit infrequently a small amount of information, called mMTC (massive MTC).

Due to the massive number of users, it is widely believed that grant-based schemes which coordinate the users — so they can use coordinated data transmission methods such as TDMA, FDMA, orthogonal CDMA, CFMA [1] and rate-splitting [2] — are ineffective due to high costs in both user energy and latency [3], [4]. Therefore, the main proposed solutions for this problem are grantless alternatives, such as “treat interference as noise” (TIN), which is implemented in practice through non-orthogonal CDMA with matched filter decoding, and slotted-ALOHA [5].

Aiming to derive fundamental results on grantless schemes, Polyanskiy introduces a novel model for the massive random access channel in [5]. In Polyanskiy’s model, only a fraction  $K_a$  of the total  $K_{\text{tot}}$  users are active at any given time and the receiver decodes only a list of  $K_a$  messages regardless of the identify of the active users. The model restricts the users to use the same (possibly randomized) codebook and the error probability is defined per user. This approach allows us to let  $K_{\text{tot}} \rightarrow \infty$  and models the random access problem well.

Using this model, Polyanskiy presents a coding theorem, which provides an achievability bound for the random access channel under finite-blocklength. Then, Polyanskiy compares the results of the mentioned practical schemes (TIN and ALOHA) to the achievable rates and shows that these schemes

G. K. Facenda and D. Silva are with the Department of Electrical and Electronic Engineering, Federal University of Santa Catarina, Florianópolis, SC 88040-990, Brazil (e-mail: gustavo.k.facenda@gmail.com; danilo.silva@ufsc.br).

are still far from the bound, in particular for a large number of active users.

Since then, research has been made seeking practical methods that approach the theoretical limits. In [6], Ordentlich and Polyanskiy present a scheme that outperforms previously practical methods for a sufficiently large ( $K_a > 150$ ) number of active users. One limitation of the method proposed in [6] is that the AWGN channel is reduced to a modulo-2<sup>ℓ</sup> AWGN channel. In [3], following [6], Vem et al. propose a scheme that allows serial interference cancellation as well as makes full use of the AWGN channel. Recently, in [7], Amalladinne et al. approach the problem through a compressive sensing point of view, presenting a low complexity compressive sensing algorithm for the massive random access channel. To the best of our knowledge, the results in [7] are the closest to the theoretical achievable rates.

In this paper, we take a different approach to improve the method in [6]. Motivated by the simplicity and good performance of coordinated orthogonal methods, we propose a practical grant-based scheme which does not assume any a priori coordination. We show that, contrary to what has been commonly assumed, the benefits of coordination can outweigh its costs, even for a massive number of users. Specifically, our scheme is able to transmit using the same number of channel uses (including the necessary feedback), while spending less energy (including base station energy) than that of [7]. Our approach for the scheduling request is to identify the transmitting users using two pieces of information: small indices randomly chosen by the users, transmitted efficiently using the scheme in [6]; and the timeslots in which each user has chosen to transmit their index. This approach significantly reduces the overhead of the scheduling request and feedback, therefore making the coordination viable.

The main contributions of our paper are summarized as follows:

- We present a model that includes both Polyanskiy’s model and certain coordinated schemes;
- We present a coordinated scheme that improves the state-of-the-art results for the Massive Random Access Gaussian Channel.

## II. PRELIMINARIES

In [8], Polyanskiy presents normal approximations for the achievable rate under finite blocklength. We use these normal approximations for the code rates in our work.

Consider the AWGN channel with input  $\mathbf{x} \in \mathbb{R}^{n_c}$  and output  $\mathbf{y} \in \mathbb{R}^{n_c}$  described by  $\mathbf{y} = \mathbf{x} + \mathbf{z}$ , where  $\mathbf{z} \sim \mathcal{N}(0, \mathbf{I})$  and  $\mathbf{x}$  is

under power constraint  $\|\mathbf{x}\|^2 \leq n_c P$ , where  $n_c$  is the length of  $\mathbf{x}$ . For this channel, an approximation of the maximum achievable rate is given by

$$R_{\text{AWGN}} \approx C_{\text{AWGN}}(P) - \sqrt{\frac{V_{\text{AWGN}}(P)}{n_c}} Q^{-1}(\epsilon). \quad (1)$$

where  $C_{\text{AWGN}}(P) = \frac{1}{2} \log_2(1 + P)$ ,  $V_{\text{AWGN}}(P) = \frac{P}{2} \frac{P+2}{(P+1)^2} \log_2^2 e$  and  $Q$  is the Q-function. This approximation is achievable using Shannon's achievability from [9].

Additionally, consider the bi-AWGN mod 2 channel with input  $\mathbf{c} \in \{0, 1\}^{n_c}$  and output  $\mathbf{y} \in [0, 2]^{n_c}$ , described by  $\mathbf{y} = (\mathbf{c} + \mathbf{z}) \bmod 2$ , where  $\mathbf{z} \sim \mathcal{N}(0, \frac{1}{4P} \mathbf{I})$ . As in [6], we use the approximation

$$R_{\text{mod2}} \approx C(P) - \sqrt{\frac{V(P)}{n_c}} Q^{-1}(\epsilon) \quad (2)$$

where

$$\begin{aligned} C(P) &= \mathbf{E}[i(\tilde{Z})] \\ V(P) &= \text{var}[i(\tilde{Z})] \\ i(\tilde{Z}) &= \log_2 \left( \frac{p_{\tilde{Z}}(\tilde{Z})}{\frac{1}{2} p_{\tilde{Z}}(\tilde{Z}) + \frac{1}{2} p_{\tilde{Z}}(\tilde{Z} - 1 \bmod 2)} \right) \end{aligned}$$

where  $\tilde{Z} = \mathbf{z} \bmod 2$ .

### III. PROBLEM STATEMENT

In this section, we first introduce the model proposed by Polyanskiy in [5]. Afterwards, we generalize it adding two extra phases – one downlink and another uplink – to the communication, allowing coordination.

#### A. Grantless Model

Let  $K_{\text{tot}} \rightarrow \infty$  be the total number of users in the network. At the beginning of a section,  $K_a$  of these users are active and wish to transmit  $k$  bits of information using the same channel. Let  $w_i \in \{1, 2, \dots, 2^k\}$  be the message of the  $i$ -th user, where  $i \in \{1, \dots, K_a\}$ , for simplicity. We assume  $w_i$  is uniform and independent across the users.

The channel is a Multiple Access Channel described as

$$\mathbf{y} = \sum_{i=1}^{K_a} \mathbf{x}_i + \mathbf{z} \quad (3)$$

where  $\mathbf{y} \in \mathbb{R}^{N_1}$  is the received signal,  $\mathbf{x}_i \in \mathbb{R}^{N_1}$  is the transmitted signal by the  $i$ -th user and  $\mathbf{z} \sim \mathcal{N}(0, \frac{N_0}{2} \mathbf{I})$ , with  $\frac{N_0}{2} = 1$ . Each transmitted signal  $\mathbf{x}_i$  is power-constrained in expectation, i.e.,  $\mathbf{E}[\|\mathbf{x}_i\|^2] \leq N_1 P_1$ .

The receiver produces a list of estimated messages denoted by  $\mathcal{L} \subseteq \{1, 2, \dots, 2^k\}$ . The probability of error, as in [5], is defined per user and regardless of the order of the messages. More precisely, the probability of error is defined as

$$\epsilon = \frac{1}{K_a} \sum_{i=1}^{K_a} \Pr[w_i \notin \mathcal{L}]. \quad (4)$$

#### B. Grant-based Model

In our model, the first phase is used for signaling activity. Using the information decoded in  $\mathbf{y}$ , the base station generates a feedback signal  $\mathbf{x}^{[f]}$  and transmits it in a broadcast channel to all users. Finally, using the information decoded from  $\mathbf{y}^{[f]}$ , the users transmit in a multiple access channel, as in the first phase.

More precisely, in our model, the transmission is split in three phases: Scheduling Request, Resource Distribution and Data Transmission. These three phases compose a session. The model for the Scheduling Request phase is the same as Polyanskiy's transmission. The Resource Distribution phase, which occurs in the downlink, is modeled as a broadcast transmission where the channel gain is the same for all users, i.e., each user  $i$  receives a signal

$$\mathbf{y}_i^{[f]} = \mathbf{x}^{[f]} + \mathbf{z}_i^{[f]} \quad (5)$$

where  $\mathbf{x}^{[f]} \in \mathbb{R}^{N_f}$  is subject to  $\mathbf{E}[\|\mathbf{x}^{[f]}\|^2] \leq N_f P_f$  and depends on  $\mathbf{y}$ , and  $\mathbf{z}_i^{[f]} \sim \mathcal{N}(0, \mathbf{I})$ .

The Data Transmission phase is modeled, again, as a multiple access channel, i.e.,

$$\mathbf{y}^{[2]} = \sum_{i=1}^{K_a} \mathbf{x}_i^{[2]} + \mathbf{z}^{[2]} \quad (6)$$

where  $\mathbf{x}_i^{[2]} \in \mathbb{R}^{N_2}$  is subject to  $\mathbf{E}[\|\mathbf{x}_i^{[2]}\|^2] \leq N_2 P_2$  and  $\mathbf{z}^{[2]} \sim \mathcal{N}(0, \mathbf{I})$ . However, this phase differs from the first phase in that each user  $i$  has side information about  $\mathbf{x}^{[f]}$  provided by  $\mathbf{y}_i^{[f]}$ .

We denote  $N = N_1 + N_2 + N_f$ . This is the total number of channel uses each user spends on this transmission. Note that the grantless model proposed by Polyanskiy is a particular case where  $N_2 = N_f = 0$  and the data is transmitted in the first phase.

#### C. Metrics of comparison

Following [5], [6] and [3], we are interested in the problem where, given some target probability of error  $\epsilon$  and number of channel uses  $N$ , we wish to minimize the energy required for the  $K_a$  users to send  $k$  bits of information. For comparison, we define the per-user metric

$$\frac{E_b}{N_0} = \frac{P_1 N_1 + P_2 N_2 + P_f N_f / K_a}{2k} \quad (7)$$

which is proportional to the total energy spent in a session. In the particular case  $N_f = N_2 = 0$ , this is the same definition used in the previous works. Note that the energy in the downlink transmission  $P_f N_f$  is used once to transmit to all users, therefore, the per-user energy in that phase is  $P_f N_f / K_a$ .<sup>1</sup>

<sup>1</sup>More generally, we want to minimize both the energy spent by the users and by the base station. However, in order to compare it with other methods, we need to define a utility function, weighing the energy used by the base station against the energy used by the users. In our understanding, the energy spent by the base station should be less significant than the energy spent by the users, therefore, a weight coefficient of "1" is conservative and is not unfair to compare against other methods.

#### IV. ORDENTLICH-POLYANSKIY SCHEME

In [6], Ordentlich and Polyanskiy present a scheme which is a special case of the  $T$ -fold ALOHA, referred to here as OP scheme. Each block of length  $N$  is split in  $V$  sub-blocks and each user chooses randomly one of the sub-blocks to transmit. Unlike ALOHA, where an error happens if two or more users transmit in the same sub-block, in OP scheme an error only happens if more than  $T$  users transmit in the same sub-block. The scheme is reviewed in the sequence.

Each active user  $i$  encodes its message  $w_i$  using a code for the Binary Adder Channel (BAC), generating the codeword  $\mathbf{c}_{\text{BAC},i} \in \{0,1\}^{k_c}$ . Observing a modulo-2 sum of  $T$  or less codewords, this code must be able to successfully recover each codeword. The particular code used is generated using the columns of the parity check matrix of a BCH code as codewords. More details can be found in [10] and [6].

Then, this codeword is encoded using a binary linear code  $\mathcal{C}$  with rate  $R_c = \frac{k_c}{n_c}$  and length  $n_c$  which is good for the bi-AWGN mod 2 channel, generating  $\mathbf{c}_i = \mathbf{c}_{\text{BAC},i} \mathbf{G} \bmod 2$ , where  $\mathbf{G} \in \{0,1\}^{k_c \times n_c}$  is the generator matrix of the linear code. This code is assumed to achieve (2). Finally, the user maps the resulting binary codeword into a real signal  $\mathbf{x}_i = 2\sqrt{PV}(\mathbf{c}_i - \frac{1}{2})$ .

Let  $\mathcal{A}_j$  be a subset of  $\{1, 2, \dots, K_a\}$  that represents the users that transmitted in the  $j$ th timeslot. For each timeslot, the receiver receives

$$\mathbf{y}_j = \sum_{i \in \mathcal{A}_j} \mathbf{x}_i + \mathbf{z}_j. \quad (8)$$

Let  $\hat{t}_j$  be a receiver's estimate of  $t_j = |\mathcal{A}_j|$ . The receiver computes

$$\mathbf{y}_{\text{CoF},j} = \left[ \frac{1}{2\sqrt{PV}} \mathbf{y}_j + \frac{\hat{t}_j}{2} \right] \bmod 2 \quad (9)$$

where the modulo 2 reduction is into the interval  $[0, 2)$  and is taken componentwise.

If  $\hat{t}_j = t_j$ , then

$$\mathbf{y}_{\text{CoF},j} = \left[ \sum_{i \in \mathcal{A}_j} \mathbf{c}_i + \tilde{\mathbf{z}}_j \right] \bmod 2 \quad (10)$$

where  $\tilde{\mathbf{z}}_j = \frac{\mathbf{z}_j}{2\sqrt{PV}}$ . Note that, since the code  $\mathcal{C}$  is linear, the sum of codewords  $\tilde{\mathbf{c}}_j = \sum_{i \in \mathcal{A}_j} \mathbf{c}_i \bmod 2$  also belongs to the code  $\mathcal{C}$ . Thus, the effective channel is

$$\mathbf{y}_{\text{CoF},j} = (\tilde{\mathbf{c}}_j + \tilde{\mathbf{z}}_j) \bmod 2. \quad (11)$$

The codeword  $\tilde{\mathbf{c}}_j$  is decoded and, since  $\tilde{\mathbf{c}}_j = \left( \sum_{i \in \mathcal{A}_j} \mathbf{c}_{\text{BAC},i} \right) \mathbf{G} \bmod 2$ , if the decoding is successful, we recover

$$\mathbf{y}_{\text{BAC},j} = \sum_{i \in \mathcal{A}_j} \mathbf{c}_{\text{BAC},i} \bmod 2. \quad (12)$$

Finally, the receiver decodes  $\mathbf{y}_{\text{BAC},j}$ , and, if the decoding is successful, it generates a list of codewords  $\mathbf{c}_{\text{BAC},i}$ ,  $i \in \mathcal{A}_j$ . These codewords are mapped into a list  $\mathcal{L}(\hat{t}_j)$  of estimated messages  $w_i$ .

On the other hand, if  $\hat{t}_j \neq t_j$ , the computation of  $\mathbf{y}_{\text{CoF},j}$  results in

$$\mathbf{y}_{\text{CoF},j} = \left[ \sum_{i \in \mathcal{A}_j} \mathbf{c}_i + (\hat{t}_j - t_j) + \tilde{\mathbf{z}}_j \right] \bmod 2. \quad (13)$$

which will likely cause an error in the decoding of the linear code if  $(\hat{t}_j - t_j) \bmod 2 \neq 0$ . If an undetected error occurs, a wrong  $\hat{\mathbf{y}}_{\text{BAC},j} \neq \mathbf{y}_{\text{BAC},j}$  is returned, which will be then decoded by the BAC code, generating a list  $\mathcal{L}(\hat{t}_j)$  of estimated messages.

For any  $\hat{t}_j$ , if a detected error occurs in any decoding step, the output is set to  $\mathcal{L}(\hat{t}_j) = \emptyset$  and an error is flagged.

The OP scheme computes a list  $\mathcal{L}(\hat{t}_j)$  for  $0 \leq \hat{t}_j \leq T$ . As argued in [6], the base station can easily choose the  $\hat{t}_j^*$  which yields the best agreement with  $\mathbf{y}_j$  and set  $\mathcal{L}_j = \mathcal{L}(\hat{t}_j^*)$ . If no list yields a good enough agreement, the decoder returns  $\mathcal{L}_j = \emptyset$ , which likely happens if an error occurs.

Finally, although we only review the modulo-2 AWGN channel, the OP scheme is extended in [6] to multi-level codes, thus the effective channel is a modulo- $2^\ell$  AWGN channel. In order to do that in a multiple access channel, the authors add a preamble in the message, which is used to align the messages in each level and recover the complete message.

Although the OP scheme achieves good results compared to previous known methods, some drawbacks can be observed. First, the reduction to a modulo- $2^\ell$  channel implies in some loss of capacity. Second, adding a preamble to the message in order to use multi-level codes causes overhead. Finally, using the BAC code allows us to recover at most  $T$  users, but there is a non-negligible probability that strictly  $t_j < T$  users transmit in the timeslot  $j$ .

Our proposed scheme handles most of these drawbacks by transmitting only a small preamble using the OP scheme, and then transmitting the message in a coordinated MAC, as described in the following section.

#### V. PROPOSED SCHEME

In this section, we describe the three phases of a session in our transmission method. In the Scheduling Request phase, instead of using the OP scheme to transmit  $k$  bits of payload, we use it to transmit a significantly smaller random identification preamble  $u$  with length  $m \ll k$ . Note, however, that this identification is not made across the  $K_{\text{tot}}$  users, but across only the  $K_a$  active users.

Additionally, we use the timeslot where each user transmitted as part of their identification, decreasing significantly the number of indices required to differentiate the  $K_a$  users. Also, note that the identification preamble is random, therefore the user symmetry of the problem is preserved.

In the Resource Distribution phase, the base station transmits a simple feedback to all users, which consists of the recovered indices concatenated, ordered by the timeslot where each index was recovered. Based on the index and its position, the users are able to identify which resources are allocated to them.

In the third phase, the users transmit information using some coordinated scheme. In principle, we can use any coordinated

orthogonal or non-orthogonal multiple access scheme for this phase. In this work, we use an orthogonal scheme, which achieves the symmetric capacity of the MAC and presents good results under finite blocklength, in particular for  $K_a \leq 100$ , as can be seen in Fig. 1 from [5]. Even for a higher number of active users, it presents a better result than other known methods. Additionally, since each user is transmitting alone in this stage, known point-to-point AWGN codes can be used, and the known finite blocklength results from [8] can be used for analysis.

In the following subsections, the three phases are described in detail.

#### A. Scheduling Request Phase

In this stage, instead of transmitting the message  $w_i$ , each user randomly picks an index  $u_i \in \{1, 2, \dots, n_p\}$ , where  $n_p = 2^m - 1$  is a design parameter, and transmits it using the OP scheme described in Section IV. We denote  $n_p$  as the length of the BCH code and  $m_p = mT$  as the length of its columns, therefore also the length of the BAC code. For the linear code, we denote  $n_{c,1}$  as the length of the code. The total number of channel uses in this stage is given by  $N_1 = n_{c,1}V$ , where  $V$  is the number of timeslots and  $n_{c,1}$  is the number of channel uses in each timeslot.

#### B. Resource Distribution Phase

After the first phase is complete, the receiver generates a feedback sequence  $(|\mathcal{L}_j|, \{u \in \mathcal{L}_j\})$ ,  $j = 1, \dots, V$ . This sequence is broadcast to the users. With that information, each transmitter knows exactly what indices were transmitted (as long as the indices were recovered), and in which timeslots. Then, each transmitter is able to identify its own index and transmit using the resource respective to the position of its index. If a user does not receive its respective index, this user does not transmit in the data transmission phase.

For example, assume that in the first timeslot, the users  $\{1, 4, 10\}$  transmitted the corresponding indices  $\{9, 3, 57\}$ . In the second timeslot, the users  $\{5, 2\}$  transmitted the indices  $\{3, 60\}$ . The base station generates the sequence  $(3, \{3, 9, 57\}, 2, \{3, 60\})$  and transmits it to the users. Although the users 4 and 5 transmitted the same index, they transmitted in different timeslots, thus they know which one is theirs. Then, for the data transmission phase, they can be ordered as follows:  $\{4, 1, 10, 5, 2\}$ .

This feedback sequence has length  $\leq \lceil \log_2(T+1) \rceil + t_j m$  per timeslot and a total feedback length  $\leq V \lceil \log_2(T+1) \rceil + K_a m$ .

#### C. Data Transmission Phase

For this phase,  $K_a$  resources (e.g. timeslots or frequency slots) are allocated to the users. Each user  $i$  that has successfully received its index in the resource distribution phase encodes its message  $w_i$  using a code for the point-to-point AWGN channel and transmits it with power  $P_2 K_a$  within their respective resource. This code is assumed to achieve (1).

In this phase, the number of channel uses is given by  $N_2 = n_{c,2} K_a$ , where  $n_{c,2}$  is the length of the code.

#### D. Error Analysis

In this section, we present an upper bound on the probability of error. Recall that the probability of error is defined per user. For the first and second phases, we make the error analysis given that the user  $i \in \{1, 2, \dots, K_a\}$  transmitted the index  $u_i \in \{1, 2, \dots, n_p\}$  in the timeslot  $j \in \{1, 2, \dots, V\}$ . For the third phase, we make the error analysis given that the user  $i \in \{1, 2, \dots, K_a\}$  transmitted the message  $w_i \in \{1, 2, \dots, 2^k\}$  using the resource  $j \in \{1, 2, \dots, K_a\}$ . By symmetry, the resulting probability of error equals the average probability of error per user.

In the first stage, four types of error can occur. First, if other  $T$  or more users choose to transmit in the timeslot  $j$ , the error event  $E_1$  occurs, with probability

$$\epsilon_1 \triangleq 1 - \sum_{t=0}^{T-1} \binom{K_a - 1}{t} \left(\frac{1}{V}\right)^t \left(1 - \frac{1}{V}\right)^{K_a - 1 - t}. \quad (14)$$

Second, if the receiver is unable to decode  $\mathbf{y}_{\text{CoF},j}$ , the error event  $E_2$  occurs, with probability upper bounded by

$$\epsilon_2 \triangleq Q\left(\left(C(P_1 V) - \frac{m_p}{n_{c,1}}\right) \sqrt{\frac{n_{c,1}}{V(P_1 V)}}\right). \quad (15)$$

This follows from the fact that, as in the OP scheme, we have a bi-AWGN mod 2 channel and the information we wish to transmit through this channel consists of  $m_p$  bits. The first stage uses power  $P_1$ , and since each user transmits only once in  $V$  timeslots, they transmit with power  $P_1 V$  during that timeslot.

A third type of error occurs when two or more users pick the same index and transmit it in the timeslot  $j$ . This leads to the error event  $E_3$  with probability upper bounded by

$$\epsilon_3 \triangleq \frac{T(T-1)}{2n_p}. \quad (16)$$

In our full paper [11], we show that this bound can be improved using an index collision resolution (ICR) method. In that case, the bound is  $\frac{K_a - 1}{V n_p}$ .

In the resource distribution phase, if the user  $i$  is unable to decode the feedback correctly, the error event  $E_f$  occurs, with probability upper bounded by

$$\epsilon_f \triangleq Q\left(\left(C_{\text{AWGN}}(P_f) - \frac{k_f}{N_f}\right) \frac{\sqrt{N_f}}{\sqrt{V_{\text{AWGN}}(P_f)}}\right) \quad (17)$$

where  $k_f = V \lceil \log_2(T+1) \rceil + K_a m$ .

In the data transmission phase, two errors can occur. First, if the receiver is unable to decode the signal transmitted in the resource  $j$ , the error event  $E_4$  happens. For this analysis we assume that (1) holds, thus the probability of error for the AWGN channel code is upper bounded by

$$\epsilon_4 \triangleq Q\left(\left(C_{\text{AWGN}}(P_2 K_a) - \frac{k}{n_{c,2}}\right) \frac{\sqrt{n_{c,2}}}{\sqrt{V_{\text{AWGN}}(P_2 K_a)}}\right). \quad (18)$$

Second, if some user  $i'$  is subject to the error  $E_f$  and it transmits using the resource  $j$  which is allocated to the user  $i$ ,

the user  $i'$  causes an error to the user  $i$ . We denote this event by  $E_{f,2}$  with probability upper bounded by

$$1 - \left(1 - \frac{\epsilon_f}{K_a}\right)^{K_a-1} \leq \frac{K_a-1}{K_a} \epsilon_f \leq \epsilon_f \triangleq \epsilon_{f,2} \quad (19)$$

where  $\frac{\epsilon_f}{K_a}$  is a loose upper bound on probability that a user  $i'$  is subject to  $E_f$  and transmits using the resource  $j$ .

Finally, the probability of error per user can be upper bounded by  $\epsilon \leq \epsilon_1 + \epsilon_2 + \epsilon_3 + \epsilon_4 + \epsilon_f + \epsilon_{f,2}$ .

## VI. OPTIMIZATION AND RESULTS

Given  $k$  and  $K_a$ , our scheme depends on the parameters  $T$ ,  $m$ ,  $n_{c,1}$ ,  $n_{c,2}$ ,  $V$ ,  $N_f$ ,  $P_1$ ,  $P_2$  and  $P_f$ . We wish to choose these parameters in order to minimize  $E_b/N_0$ , subject to constraints on  $\epsilon$  and  $N$ . In order to simplify the optimization, we use<sup>2</sup>  $P_1 = P_2 = P_f/K_a = P$ . In this case, since  $N$  and  $k$  are fixed, minimizing  $E_b/N_0 = \frac{PN}{2k}$  is equivalent to minimizing  $P$ .

This optimization is hard in general since all variables, except  $P$ , are integers. Thus, we follow a heuristic approach. Given a fixed  $T \in \mathbb{Z}$ , we relax the integer constraints on the remaining variables, finding their optimal values over  $\mathbb{R}$ . This is possible since all the error expressions are computable with real variables (except  $\epsilon_1$  with  $T$ ). The resulting value of  $P$  gives a lower bound on the achievable  $P$ . Then, for each variable in the sequence  $m$ ,  $n_{c,1}$ ,  $n_{c,2}$ ,  $V$ , its optimal relaxed value is rounded to  $\mathbb{Z}$  and kept fixed, followed by a new run of the relaxed optimization for the remaining variables. We use rounding to the nearest integer, except in the case of  $m$ , where we compare floor and ceiling, choosing the one that yields the smallest  $P$ . After these variables are fixed, we choose  $N_f = N - N_1 - N_2$  and compute the required power  $P$  to achieve the target error  $\epsilon$ . This process gives an achievable  $P$  for the initially chosen  $T$ . Finally, this process is repeated for  $T = 1, \dots, T_{\max}$  to find the smallest  $P$ .

While this approach is not guaranteed to be optimal, it has shown experimentally to yield values of  $P$  very close to the relaxed lower bound (ratio below 0.06 dB for all cases tested).

As in [3], [6], [7], we use  $k = 100$ ,  $N = 30000$  and  $\epsilon = 0.05$ . The results are presented in Fig 1 and compared to [3], [6], [7]. We also plot the random coding bound and the orthogonal MA bound [5], where perfect coordination is assumed a priori. As can be seen, even without ICR, our method outperforms that in [7] for a moderate number of users ( $K_a \leq 150$ ), but its performance comparatively worsens as  $K_a$  grows. With ICR, our method outperforms [7] significantly for all the values of  $K_a$  tested.

## VII. CONCLUSION

We presented a new scheme for the Massive Random Access Gaussian channel which allows coordinating the users for a better overall performance at the low cost of a short broadcast feedback from the base station.

We presented an error analysis, which allows simple design and optimization of the scheme, and showed that our method

<sup>2</sup>For the scenarios considered here, this simplification is found experimentally to have negligible impact on performance.

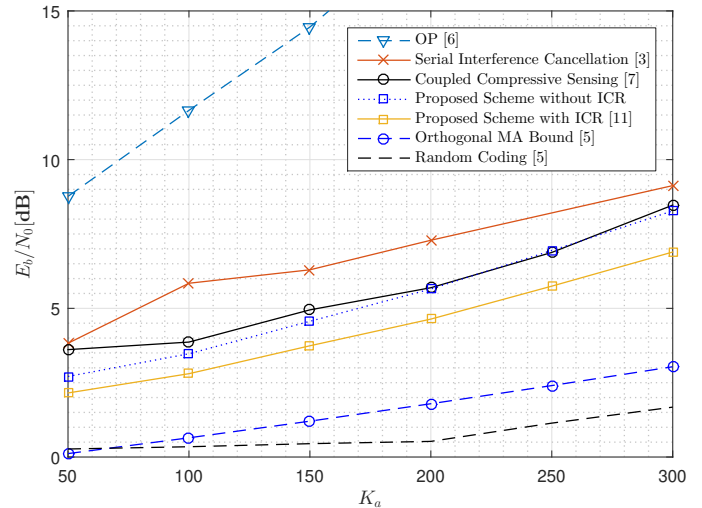


Fig. 1. Comparison between the  $E_b/N_0$  required for  $k = 100$  bits,  $N = 30000$  channel uses,  $\epsilon = 0.05$ .

outperforms the state-of-art in [7], as well as the practical schemes in [6] and [3], as long as a short feedback can be transmitted from the base station.

As in other works in this area of research, we use finite-blocklength results in order to obtain achievability bounds. The code design for the short blocklengths involved is an important open problem.

## REFERENCES

- [1] J. Zhu and M. Gastpar, "Gaussian multiple access via compute-and-forward," *IEEE Transactions on Information Theory*, vol. 63, no. 5, pp. 2678–2695, May 2017.
- [2] B. Rimoldi and R. Urbanke, "A rate-splitting approach to the Gaussian multiple-access channel," *IEEE Transactions on Information Theory*, vol. 42, no. 2, pp. 364–375, March 1996.
- [3] A. Vem, K. R. Narayanan, J. Cheng, and J. Chamberland, "A user-independent serial interference cancellation based coding scheme for the unsourced random access Gaussian channel," in *2017 IEEE Information Theory Workshop (ITW)*, Nov 2017, pp. 121–125.
- [4] C. Bockelmann, N. Pratas, H. Nikopour, K. Au, T. Svensson, C. Stefanovic, P. Popovski, and A. Dekorsy, "Massive machine-type communications in 5g: physical and MAC-layer solutions," *IEEE Communications Magazine*, vol. 54, no. 9, pp. 59–65, Sep. 2016.
- [5] Y. Polyanskiy, "A perspective on massive random-access," in *2017 IEEE International Symposium on Information Theory (ISIT)*, June 2017, pp. 2523–2527.
- [6] O. Ordentlich and Y. Polyanskiy, "Low complexity schemes for the random access Gaussian channel," in *2017 IEEE International Symposium on Information Theory (ISIT)*, June 2017, pp. 2528–2532.
- [7] V. K. Amalladinne, A. Vem, D. K. Soma, K. R. Narayanan, and J. Chamberland, "A coupled compressive sensing scheme for unsourced multiple access," in *2018 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP)*, April 2018, pp. 6628–6632.
- [8] Y. Polyanskiy, H. V. Poor, and S. Verdú, "Channel coding rate in the finite blocklength regime," *IEEE Transactions on Information Theory*, vol. 56, no. 5, pp. 2307–2359, May 2010.
- [9] C. E. Shannon, "Probability of error for optimal codes in a Gaussian channel," *The Bell System Technical Journal*, vol. 38, no. 3, pp. 611–656, May 1959.
- [10] I. Bar-David, E. Plotnik, and R. Rom, "Forward collision resolution-a technique for random multiple-access to the adder channel," *IEEE Transactions on Information Theory*, vol. 39, no. 5, pp. 1671–1675, Sept 1993.
- [11] G. K. Facenda and D. Silva, "Efficient scheduling for the massive random access Gaussian channel," May 2019, submitted to *IEEE Transactions on Wireless Communications*. [Online]. Available: <https://arxiv.org/abs/1905.06479>