A study of semi-blind and blind equalization systems

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***Resumo*— Este artigo é um estudo introdutório a algumas técnicas de equalização. Nós abordamos a técnica da equalização semi-cega e comentamos uma introdução à técnica de equalização cega. Nós analisamos o desempenho dessas técnicas usando os algoritmos adaptativos LMS, RLS e CMA.**

***Palavras-Chave*— equalização semi-cega, equalização cega, LMS, RLS, CMA**

***Abstract*— This article is an introductory study of some equalization techniques. We approach the technique behind the semi-blind equalization and comment an introduction to the blind equalization technique. We analyze the prosecution of these techniques using the LMS, RLS and the CMA adaptive algorithms.**

***Keywords*— semi-blind equalization, blind equalization, LMS, RLS, CMA**

I. INTRODUCTION

Considering the classical model of a digital communication system, information transmitted over a channel with memory is corrupted by ISI and noise. In order to recover the original signal, an equalizer filter is used in the receiver [1].

The main objective of the equalizer is to estimate the inverse system of the channel and perform the deconvolution of the transmitted signal, reducing the interference. Then the equalizer starts the *acquisition mode*, where the transmitter emits training symbols and the receiver recovers a delayed training symbol version. After the convergence of the adaptive algorithm in the acquisition mode to a level of lower interference, the equalizer switches to the tracking mode, for the symbol estimation error is low. This type of equalization is known as *adaptive equalization*. However, if the equalizer can extract the transmitted information from the received signal without any training, the procedure is known as *blind equalization* [2], [3] . Intermediately, there is a strategy where the training of the equalizer finishes, ignoring if the acquisition mode has finished or not. Consequently, the tracking mode starts prematurely, being able (or not) to converge at a lower level of interference. This process is known as *semi-blind equalization*.

II. ADAPTIVE EQUALIZATION TECHNIQUES

Equalizers can be modelled as transversal filters that self-adjust their coefficients according to an adaptive algorithm.

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The *Least-Mean Square (LMS)* algorithm, for example, minimizes the mean-square error of the estimation error and updates the filter's coefficients, which are updated by the following equation:

$$\mathbf{w}(n+1) = \mathbf{w}(n) + \mu \mathbf{u}(n) e^*(n) \tag{1}$$

$\mathbf{w}(n)$ is the coefficient vector at time n, $\mathbf{u}(n)$ is the tap-input data vector, $e(n)$ is the estimation error and μ is the step parameter, which has to obey some conditions in order to make the LMS algorithm converge [4].

Until now, we have obtained the best solution for the adaptive equalization problem using the Wiener filters. They use ensemble averages, assuming a wide-sense stationary enviroment, to perform calculations. However, it is also possible to model the adaptive equalization problem as a linear least-squares problem [4]. Instead of using all realizations of the operational enviroment, this method uses *time-averages*. Therefore the filter depends on the quantity of samples used. A recursive algorithm using the least-squares method was devised to minimize the estimation error. This algorithm is known as *Recursive Least-Squares (RLS)*, known for its fast convergence and computational complexity. [4].

III. BLIND AND SEMI-BLIND TECHNIQUES

In the semi-blind equalization, first of all, the receiver generates a delayed *training symbol* $s(n-L)$ which represents the best symbol for recover the emitted symbol. The estimation error in the acquisition mode is $e(n) = y(n) - s(n-L)$, where $y(n)$ is the equalizer output. Then applying an adaptive algorithm, the receiver converges to a lower level of interference. In this moment, the actual data transmission begins and now the equalizer switches to the *decision directed mode* [5]. The estimation error in this mode is $\tilde{e}(n) = y(n) - \tilde{s}(n)$, where $\tilde{s}(n)$ is the last decided symbol. If the signal's properties change drastically, the equalizer will have to be reinitialized.

On the other hand, there is a class blind equalization techniques known as *Bussgang algorithms* [6], [7]. The estimation error is generated using a memoryless non-linear system $\Gamma(n)$ as a reference. Therefore we denote the error as $e(n) = y(n) - \Gamma(n)$. Depending on the way $\Gamma(n)$ is chosen, a blind adaptive algorithm is selected. An important Bussgang algorithm is the *Godard Constant Modulus Algorithm* (CMA) where $\Gamma(n) = y(n) - y(n)\left[|y(n)|^2 - R\right]$ [2]. In this case, R is a constant that depends on the adopted modulation.

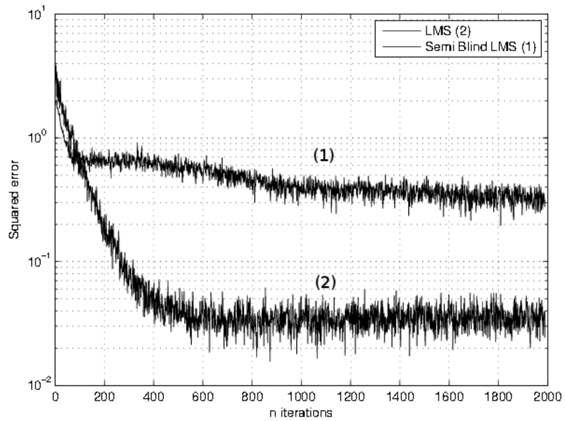

Fig. 1. Learning curve of the LMS and semi-blind LMS equalizers. Switched in n = 100.

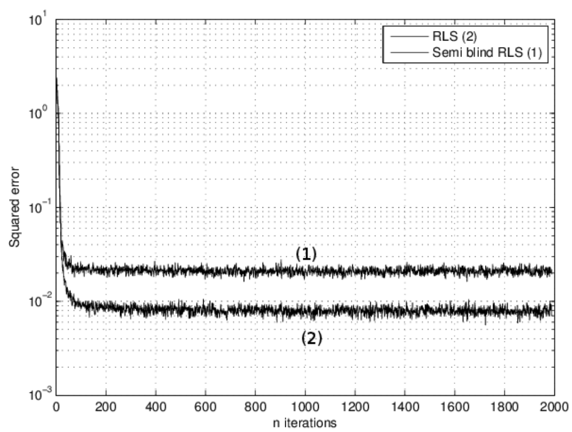

Fig. 2. Learning curve of the RLS and semi-blind RLS equalizers. Switched in n = 20.

IV. Computational Experiments

We carried out three experiments using linear equalizers to compare the performance of some adaptive algorithms: LMS, RLS and CMA. We also compared the supervised and the semi-blind equalization techniques using the LMS and the RLS algorithm. The performance criterion used is the analysis of their learning curve. We transmitted 2000 symbols using a 4-PAM modulation and the SNR was set to 25 dB. The impulse response of the channel is described by the raised cosine defined as

$$h(n) = \begin{cases} \frac{1}{2}\left[1 + cos(\frac{2\pi}{W}(n-2))\right] & n = 1,2,3 \\ 0 & \text{otherwise} \end{cases}$$

where $W = 2$ is a parameter that controls amplitude distortion produced by the channel. Both equalizers have $M = 11$ tap-weights.

The first experiment illustrated the learning curves of the supervised LMS and the semi-blind LMS in figure (1). We used 50 symbols to train the semi-blind LMS an the step-size parameter was 0.001.

In the second experiment we compared the supervised RLS and the semi-blind RLS. The RLS algorithm converges faster than LMS, therefore the equalizer can acquire more confiability in switching from the acquisition mode to the decision directed mode using less symbols. Then we used 20 training symbols in this simulation. The result is depicted in figure (2).

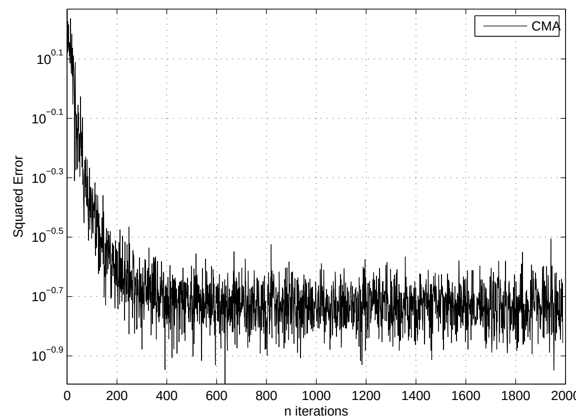

Fig. 3. Learning curve of the CMA equalizer.

The third experiment, described in the figure (3), is the simulation of the CMA, a blind equalization algorithm. In this simulation, we used $R = 2$ for the 4-PAM modulation. Since it does not require a reference signal, it is more suitable for practical applications.

V. Conclusions

The semi-blind equalization technique is a classical procedure that has a training mode faster and shorter than the adaptive equalizer's. Its convergence and performance properties relies mainly on the chosen adaptive algorithm and the quantity of training symbols. As we increase the training symbols, the equalizer gains more confiability. In order to vanish the necessity of training mode, blind algorithms were developed. They do not require a reference signal, but their squared error usually is higher.

References

[1] J. C. Mota, *Equalização Adaptativa em Sistemas Ruído-Digitais: das Técnicas em frequência à Teoria da desconvolução autodidata*, Tese de Doutorado, DECOM/FEE/UNICAMP, 1992.
[2] Carlos A. F. da Rocha, João M. T. Romano, *Equalização autodidata: fundamentos, novas propostas e perspectivas*, Revista Brasileira de Telecomunicações,Volume 10, numero 1, Dezembro de 1995.
[3] C. A. F. da Rocha, *Técnicas Preditivas para Equalização Autodidata*, Tese de Doutorado,DECOM/FEE/UNICAMP, 1996.
[4] Simon S. Haykin, *Adaptive filter theory*, Prentice Hall, 3ed, 1996.
[5] J. M. T. Romano et al, *Unsupervised Signal Processing*, CRC Press, 1ed, 2011.
[6] A. Belouchrani, K. Abed-Meraim, J.-F. Cardoso and E. Moulines. *A blind source separation technique using second-order statistics. IEEE Transactions on Signal processing*, February, 1997.
[7] S. Bellini. *Bussgang Techniques for blind deconvolution and Equalization*, Prentice Hall, Englewoord Cliffs, NJ, 1994.
[8] C.Y.Chi, C.H. Chen and C.Y. Chen. *Blind equalization and system identification: batch processing algorithms, performance and applications*. Springer, London, U.K. 2006.
[9] Qureshi, S.U.H.; , "*Adaptive equalization*," Proceedings of the IEEE , vol.73, no.9, pp. 1349- 1387, Sept. 1985.