A General Approach Analysis for Amplify-and-Forward Relaying in Cooperative Diversity Networks

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Abstract—A general approach for analyzing the performance of dual-hop cooperative diversity networks with nonregenerative amplify-and-forward (AF) relays and employing maximal-ratio combining (MRC) at the destination is proposed. Either variable or fixed gain relaying systems can be investigated from our methodology. Closed-form approximations for the outage probability (OP) and average symbol error rate (SER) of linear combinations are introduced. Our formulations are simple and yield results instantaneously, regardless the number of relays used, in contrast to the exact solution which is indeed intricate and difficult to evaluate especially when the number of relays increases. The inherent simplicity of our method makes it attractive to serve as a benchmark in the design and performance evaluation of cooperative networks with variable-gain/fixed-gain relays and employing MRC. The accuracy of our approximations is illustrated through Monte Carlo simulation and insightful discussions are provided.

Keywords—Amplify-and-forward relaying, cooperative diversity, dual-hop transmissions, moment-based estimators, performance analysis.

I. INTRODUCTION

With the continuous necessity for higher throughput and increased data rates in wireless communication systems, the concept of cooperative diversity has been recently gaining considerable interest [1]–[6]. The key idea is that, in addition to the direct signal from the source to the destination, multiple cooperative nodes (relays) collaborate together to relay the signal of the source node to the destination node. As a result, the latter can receive multiple independent copies of the same signal, and can hence achieve diversity without the need to implement multiple antennas at the source or destination nodes. Many other benefits can also be attained from this strategy, e.g., expansion of the coverage without using high power levels at the source, increase of connectivity, higher capacity, etc.

Depending on the nature and complexity of the relaying technique, cooperative diversity networks can be broadly categorized as either nonregenerative or regenerative. In the former, the relays simply amplify and forward (AF) the received signal, while in the latter the relays decode, encode, and then forward the received signal to destination. The AF mode puts less processing burden on the relays and, hence, is often preferable when complexity and/or latency are of importance. This paper primarily focuses on AF relaying systems.

Concerning the performance analysis of dual-hop cooperative networks employing arbitrary AF relays, several works are already available in the open literature. More specifically, a lower bound for the average bit error rate of collaborative dual-hop transmissions was presented in [7], where the results were expressed in terms of Meijer G-functions. In [8], an asymptotic expression for the average symbol error rate (SER) of AF relaying systems was derived. Performance bounds for cooperative diversity networks over Nakagami-$m$ fading channels were also obtained in [9]. In [10], an approximate expression for the SER of selection AF relaying systems was provided. In [11] and [12], the authors derived closed-form expressions for the outage probability for Rayleigh and Nakagami-$m$ channels, respectively with decode-and-forward relays.

In this paper, different previous works, we propose a general approach for analyzing the performance of dual-hop cooperative diversity networks with an arbitrary number of relays, undergoing Rayleigh fading, and employing MRC at the destination. Assuming either variable-gain or fixed-gain relaying, closed-form approximations for the outage probability (OP) and average SER are presented. The simplicity of our formulations provides a much better computational efficiency than the exact solution, which is indeed intricate and difficult to evaluate especially when the number of relays increases. In addition, our approximate expressions are obtained instantaneously, regardless the number of relays employed. This is a considerable advantage when compared to Monte Carlo
Fig. 1. A dual-hop cooperative diversity network with $M$ relays and a direct link.

simulation, since that in the latter case, although results are reliable, the plotting time can be exhaustive.

The remainder of this paper is structured as follows. In Section II, the system and channel models are described. In Section III, our approach is presented and a closed-form approximate expression for the OP is derived. Such expression holds for either variable or fixed gain relays as well as MRC techniques at the destination. Section IV extends our methodology to the analysis of the average SER of linear modulations, and a closed-form approximate expression for this performance metric is derived. Section V compares our results to those obtained through Monte Carlo simulation. Finally, the main conclusions of this Letter are summarized in Section VI.

II. SYSTEM AND CHANNEL MODELS

Consider the wireless network illustrated in Fig. 1, where the source node ($S$) communicates with the destination node ($D$) not only directly but also through $M$ cooperative nodes, namely relays ($R_1, \ldots, R_M$). These latter assist in the transmission by providing multiple copies of the original signal to node $D$. Also, it is assumed that the AF mode, with either variable or fixed gain, is applied. As previously mentioned, such a mode is less complex and easier to implement compared to the decode-and-forward one.

In order to guarantee orthogonal transmission, a time-division channel allocation scheme, with $M+1$ time slots, is used. In the first time slot, node $S$ broadcasts its signal to node $D$ and to the set of $M$ relay nodes. During the following $M$ time slots, the relays amplify the signals received from node $S$ and forward them to node $D$. This transmission protocol is described in detail in [4]. The mutually independent complex channel gains between the nodes, represented by $h_{SD}$, $h_{SR_i}$, and $h_{DR_i}$, $i = 1, \ldots, M$, are modeled as zero-mean circularly symmetric Gaussian random variables (RVs). Hence, their respective instantaneous signal-to-noise ratios (SNRs), $\gamma_{SD}$, $\gamma_{SR_i}$, and $\gamma_{DR_i}$, are exponentially distributed RVs. Then, all the channels ($S-D$, $S-R_i$, $R_i-D$) experience Rayleigh flat fading with average SNRs $\bar{\gamma}_{SD}$, $\bar{\gamma}_{SR_i}$, and $\bar{\gamma}_{DR_i}$. We denote by $\gamma_i$ the equivalent SNR of the relaying channel associated with $R_i$ ($S-R_i-D$). Without any loss of generality, we also assume here that all additive white Gaussian noise (AWGN) terms have zero mean and equal variance.

In the cooperative system described above, consider that MRC is employed at node $D$. In such scheme, the received signals are cophased, each signal is amplified appropriately for optimal combining, and the resultant signals are added. Hence, the instantaneous SNR at the combiner’s output, $\gamma_{MRC}$, is given by

$$\gamma_{MRC} = \gamma_{SD} + \sum_{i=1}^{M} \gamma_i. \quad (1)$$

For the case with variable-gain relays, the cumulative distribution function (CDF) of $\gamma_i$ can be obtained from [13, Eq. 27] according to

$$F_{\gamma_i}(\gamma) = 1 - \frac{2\gamma}{\sqrt{\gamma_{SR_i}\gamma_{RD}}} K_1\left(\frac{2\gamma}{\sqrt{\gamma_{SR_i}\gamma_{RD}}}\right) \times \exp\left(-\gamma\left(\frac{1}{\gamma_{SR_i}} + \frac{1}{\gamma_{RD}}\right)\right), \quad (2)$$

where $K_1(\cdot)$ is the first-order modified Bessel function of the second kind [14, Eq. 9.6.22]. Using the approximation $K_1(z) \approx 1/z$ [14, Eq. 9.6.9], (2) can be approximated as

$$F_{\gamma_i}(\gamma) \approx 1 - \exp\left(-\gamma\left(\frac{1}{\gamma_{SR_i}} + \frac{1}{\gamma_{RD}}\right)\right), \quad (3)$$

which corresponds to the CDF of an exponential distribution with average SNR $1/\gamma_{EQ_i} = 1/\gamma_{SR_i} + 1/\gamma_{RD}$. Considering now the case with fixed-gain relays, the CDF of $\gamma_i$ can be expressed as [15, Eq. 9]

$$F_{\gamma_i}(\gamma) = 1 - 2\left(1 - \frac{C}{\gamma_{SR_i}\gamma_{RD}}\right) K_1\left(2\frac{C}{\gamma_{SR_i}\gamma_{RD}}\right) \times \exp\left(-\gamma\left(\frac{1}{\gamma_{SR_i}} + \frac{1}{\gamma_{RD}}\right)\right), \quad (4)$$

where $C$ is a constant given by $C = 1/G^2 N_{SR_i}$, with $G$ denoting the gain of the relay, and $N_{SR_i}$ being the mean power of the AWGN signal pertaining to the link $S-R_i$. Note that the upper bound in (4) can be approximated by (3) making use of $K_1(z) \approx 1/z$ [14, Eq. 9.6.9].

III. OUTAGE PROBABILITY ANALYSIS

An important performance measure in wireless cooperative networks is the outage probability (OP). Under MRC, such a metric is defined as the probability that the combiner output SNR, $\gamma_{MRC}$, falls below a predefined threshold, $\gamma_{th}$, and can be expressed as [16]

$$P_{\gamma_{MRC}}(\gamma_{th}) = \int_{0}^{\gamma_{th}} P_{\gamma_{MRC}}(\gamma)d\gamma, \quad (5)$$

$$P_{\gamma_{MRC}}(\gamma) = 1 - \frac{C}{\gamma_{SR_i}\gamma_{RD}} K_1\left(2\frac{C}{\gamma_{SR_i}\gamma_{RD}}\right) \times \exp\left(-\gamma\left(\frac{1}{\gamma_{SR_i}} + \frac{1}{\gamma_{RD}}\right)\right), \quad (4)$$

where $C$ is a constant given by $C = 1/G^2 N_{SR_i}$, with $G$ denoting the gain of the relay, and $N_{SR_i}$ being the mean power of the AWGN signal pertaining to the link $S-R_i$. Note that the upper bound in (4) can be approximated by (3) making use of $K_1(z) \approx 1/z$ [14, Eq. 9.6.9].
where $f_{\gamma_{\text{MRC}}}^{\text{MRC}}(\cdot)$ is the probability density function (PDF) of $\gamma_{\text{MRC}}$. Note that (5) is indeed the CDF of $\gamma_{\text{MRC}}$ evaluated at $\gamma_{0}$. In general, for MRC receivers, the exact solution of (5) involves multifold integrals of PDFs or integral of the product of moment generating functions (MGFs), certainly non-attractive especially when the number of relaying links increases. In fact, beyond $M = 4$ cooperative relays, these solutions become computationally impractical. For instance, for $M = 4$ and using MATHEMATICA a plot might take more than one hour to generate, in addition to the fact the results may not even converge. In this case, the exact solution can be obtained in a reliable manner only through Monte Carlo simulations. But, even with this approach, the plotting time can be exhaustive and increases as the number of relays is higher. Therefore, it is certainly of interest to find simple close-form approximations that can be used as an alternative and to circumvent these drawbacks. As will be seen next, our approximations are simple irrespective of the number of relays employed in the cooperation.

Our approach follows the same rationale for both types of AF relay gains (variable or fixed). Hereafter, we consider the variable-gain relaying case. However, some comments will also be provided for the fixed-gain scenario. Firstly, the equivalent envelope statistics of the relaying link $(S - R_{i} - D)$, denoted $R_{i}$, is approximated by a Rayleigh distribution, as performed in (3). Doing so, all the received equivalent envelopes at node $D$ become Rayleigh distributed with average SNRs $\bar{\gamma}_{SD}$ (link $S - D$) and $\bar{\gamma}_{EQ_{i}}$ (link $S - R_{i} - D$). The second step consists in using the statistics of the Nakagami-$m$ distribution for approximating the CDF of $\gamma_{\text{MRC}}$. This is motivated by the validity of approximating the sum statistics of Nakagami-$m$ RVs through a Nakagami-$m$ distribution [17], [18]. Hence, as the Rayleigh distribution is a special case of the Nakagami-$m$ one, it is expected that the sum statistics of Rayleigh RVs can be well-modeled by a Nakagami-$m$ distribution. Then, based on the above, our proposal is to approximate the exact solution of (5) by the OP of a single Nakagami-$m$ RV [16], i.e.,

$$F_{\gamma_{0}}^{\text{MRC}}(\cdot) \approx 1 - \frac{\Gamma(m, m \gamma_{0}/\bar{\gamma})}{\Gamma(m)}$$

(6)

where $\Gamma(\cdot)$ is the Gamma function [19, Eq. 8.310.1], $\Gamma(\cdot, \cdot)$ is the incomplete Gamma function [19, Eq. 8.350.2], $m$ is the Nakagami fading parameter, and $\bar{\gamma}$ is the average SNR. It should be emphasized that (6) applies for both types of AF relays. Note that, for the fixed-gain case, the expression in (6) makes sense a priori once the upper bound in (4) can be approximated by the CDF of an exponential distribution. Interestingly, although the approximations for these kind of relays are derived from upper bounds, in most of the cases plots show that the accuracy is better than that employing variable-gain relays, as will be attested shortly from the numerical results.

In order to render (6) a reasonable approximation, we shall use moment-based estimators for the computation of the required parameters, $m$ and $\bar{\gamma}$, in terms of the exact moments of the envelope of the Nakagami-$m$ distribution. Let $R_{N}$ be the envelope of such distribution and, for the moment, knowledge of $E(R_{N}^{2})$ and $E(R_{N}^{4})$, with $E(\cdot)$ denoting expectation. Moment-based estimators for $m$ and $\bar{\gamma}$ can be obtained from [18, Eq. 4 and 5] according to:

$$\bar{\gamma} = E(R_{N}^{2})$$

(7a)

$$m = \frac{E^{2}(R_{N}^{4})}{E^{2}(R_{N}^{2}) - E^{2}(R_{N}^{2})}$$

(7b)

It remains to find $E(R_{N}^{2})$ and $E(R_{N}^{4})$, which were assumed to be known. In the sequel, expressions for these arbitrary moments are presented.

For MRC, as the output envelope consists of the sum of squared envelopes, the arbitrary moments of $R_{N}$ can be obtained using

$$E(R_{N}^{2k}) = \sum_{k_{2}=0}^{k} \sum_{k_{1}=0}^{k_{2}} \cdots \sum_{k_{M-1}=0}^{k_{M-2}} \binom{k}{k_{2}} \binom{k_{2}}{k_{1}} \cdots \binom{k_{M-2}}{k_{M-1}} \times E(R_{SD}^{2k_{2}}) E(R_{EQ_{1}}^{2k_{1}}) \cdots E(R_{EQ_{M}}^{2k_{M-1}}),$$

(8)

where $R_{SD}$ and $R_{EQ_{i}}$ denote the envelope of links $S - D$ and $S - R_{i} - D$, respectively. The Rayleigh marginal moments required in (8) are given by

$$E(R_{i}^{n}) = \Gamma(1 + n/2, \bar{\gamma}_{EQ_{i}}/\bar{\gamma}).$$

(9)

It is noteworthy that the closed-form approximation in (6) can be used when either variable or fixed gain relays are used. In [15], it was shown that the exact curves for systems employing these two types of relays are very close, leading us to conjecture that such an approximation will perform well for both kinds of relays. As will be seen, the approximations for fixed-gain relays outperform those for variable-gain ones.

IV. AVERAGE SYMBOL ERROR RATE

Another performance criterion frequently used in wireless cooperative networks is the average SER. For linear modulations, such metric is given by [16]

$$P_{e} = \text{Pr}(Y > \sqrt{2\alpha \gamma_{0}})$$

(10)

where $Q(x) = \frac{1}{\sqrt{2\pi}} \int_{x}^{\infty} \exp(-t^{2}/2) dt$, $\alpha$ is a constant depending on the modulation format, and $\gamma_{0}$ is the SNR at the output of the MRC combiner. Performing a similar procedure to that in [10], (10) can be rewritten as

$$P_{e} = \text{Pr}(\gamma_{\text{MRC}} < Y^{2}/\alpha)$$

$$= E_{Y} \left( F_{\gamma_{\text{MRC}}} \left( \frac{Y^{2}}{\alpha} \right) \right),$$

(11)

where $Y$ denotes a RV with standard normal distribution. Then substituting (6) into (11), and following the well-established statistical procedure for the calculation of expectations, the average SER can be approximated by

$$P_{e} \approx \int_{0}^{\infty} \left( 1 - \frac{\Gamma(m, m y^{2}/\alpha \bar{\gamma})}{\Gamma(m)} \right) \frac{1}{\sqrt{2\pi}} \exp \left( \frac{-y^{2}}{2} \right) dy$$

$$\approx \frac{1}{2} - \frac{\alpha \bar{\gamma}}{2m \pi} \Gamma \left( \frac{3}{2} + m, \frac{\pi}{\alpha \bar{\gamma}} \right)$$

(12)
where $\gamma$ is the Gaussian hypergeometric function. Eq. (12) can be used when either variable or fixed-gain relays are employed. Also, the parameters $m$ and $\gamma$ required in this expression can be obtained from (7), whereas the arbitrary moments are calculated from (8).

V. NUMERICAL RESULTS AND DISCUSSIONS

In this Section, our analytical results, (6) and (12), are plotted and compared with the exact results obtained through Monte Carlo simulations. For the average SER plots, due to space limitation, we only provide those for the case with variable-gain relays, which actually result in worse accuracy than the fixed-gain scenario. In the following, we consider phase-shift keying (PSK) modulation, which is given by setting $\alpha = 2$ in (12). The number of relays $M$ varies from 1 to 5 and the links $S – R_i$ and $R_i – D$ are assumed to be independent and identically distributed. In the results corresponding to relays with fixed gain, we consider, without any loss of generality, $C = 1$.

Figs. 2 and 3 plot the OP as a function of the threshold level $\gamma_{th}$ (dB). The links, $S – R_i$ and $R_i – D$, have average SNRs equal to 5dB, whereas the link $S – D$ is assumed to have an average SNR of 0dB. In general, for the variable-gain relaying case, note that the proposed approximations lose their tightness for high threshold values and as $M$ increases. Such a fact occurs mainly due to the approximation of the equivalent channel $S – R_i – D$ by a Rayleigh channel, which is performed by considering the term $2\gamma_{th}/\sqrt{\gamma_{SR_i} \gamma_{R_iD}}$ to be very close to zero, i.e., for low values of $\gamma_{th}$. It is then expected that for high threshold levels, the approximations will not perform well. In addition, as $M$ increases there is an accumulation of errors because more equivalent channels $S – R_i – D$ will be approximated by a Rayleigh channel. Concerning the slight inaccuracies observed in some curves at low threshold levels for these kind of relays, they result from the approximation of the sum of Rayleigh envelopes and powers by the statistics of a Nakagami-$m$ distribution. For the fixed-gain relaying case, it can be observed that the approximations outperform those of the variable-gain case at high threshold levels. This is because of the exact OP for the former being lower than for the latter at high values of $\gamma_{th}$, as shown in [15] for the non-diversity case. Thus, as our approximate curves are the same for both kinds of relays, it is expected that, based on the behavior of their respective exact solutions, the approximations are more precise in the fixed-gain case compared to the variable-gain one.

Figs. 4 illustrates the average SER against the average SNR, $\bar{\gamma}$, at the destination’s output when assuming $\gamma_{SR_i} = \gamma_{R_iD} = 3\gamma_{SD}$ and using variable-gain relays. As a whole, when $\bar{\gamma}$ increases, the accuracy of the approximations improves once such statistics are directly proportional to the average SNRs of the links $S – R_i$ and $R_i – D$. Hence, increasing the average SNR at the combiner’s output means that $\bar{\gamma}_{SR_i}$ and $\bar{\gamma}_{R_iD}$ increase, thus yielding a decrease of the term $2\gamma_{th}/\sqrt{\gamma_{SR_i} \gamma_{R_iD}}$. Similar to the OP and justified by the same reasons as well, our approximations loose from their tightness to the exact solutions when $M$ increases or when the values of $\bar{\gamma}$ get higher. For the fixed-gain relaying case, although no curves were shown here, due to space limitation as aforementioned, validations of the corresponding results were performed and it can be confirmed that our approximations perform better than those for the variable-gain scenario. Such behavior was already attested when the OP curves were sketched.

To conclude this section, a remark is in order, especially when considering variable-gain relays. We would like to emphasize that, although our approximations are not as tight in high threshold regions or, equivalently, in low average output SNR ranges, their inherent computational simplicity renders them very attractive for the performance analysis of cooperative diversity systems employing MRC techniques. Indeed, from a practical viewpoint, a first reasonable and
fast assessment of performance can be done by using the formulations presented here. Note that the results are obtained instantaneously, regardless of the number of relays employed. Furthermore, our approach can be applied as a benchmark for the design and performance evaluation of cooperative networks with AF relays and employing MRC.

VI. CONCLUSIONS

In this work, a general and simple approach for analyzing the performance of dual-hop cooperative diversity networks employing nonregenerative amplify-and-forward relays, with variable or fixed gain, was proposed. Closed-form approximations for the outage probability and average SER were obtained. Our unified analysis considers unrestrictedly the use of MRC techniques at the destination node. It is noteworthy that the proposed approach allows for a practical way to evaluate the performance of these kind of systems, and serve as a powerful tool for the design of communication protocols and algorithms for such cooperative networks.

REFERÊNCIAS