Admissible number of users in OFDM/TDMA based wireless systems through loss probability estimation considering multifractal traffic modeling

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Abstract – In this paper, we present an admission control scheme for an OFDM (Orthogonal Frequency-Division Multiplexing)/TDMA(Time Division Multiplexing Access) based wireless system taking into account the multifractal behaviour of the traffic flows. To this end, first, we show evidences of multifractal characteristics on wireless network traffic traces. These findings motivated us to derive a data loss probability expression that can be applied to the OFDM/TDMA based wireless system. Moreover, the loss probability expression is used to control the admission of new users to the OFDM/TDMA system. Simulations and comparisons to other methods are carried out in order to verify the performance of the proposed loss probability estimation approach and the efficiency of the admission control scheme in accepting new users to the OFDM/TDMA system.

Keywords – Loss probability, Multifractal Analysis, Admission Control Scheme.

I. INTRODUCTION

Numerous traffic models have been developed for telecommunication networks [1]. Among them, we can include Markov-based models, autoregressive models, self-similar models and multifractal models [2]. Such traffic models provide ways of characterizing network traffic behaviour through analytical techniques or simulation.

Self-similar and multifractal models have been receiving great attention due to their analysis and modeling performance related to real network traffic. In fact, some researches have revealed that multifractal models are adequate in describing some network traffic characteristics [3],[4],[5],[6],[7].

Wireless communication systems are designed to support a diverse range of services and applications. Due to their inner characteristics, wireless LAN traffics are typically affected by non-ideal channel condition underlying MAC protocol and user mobility [2],[8],[9]. Therefore, efficient QoS (Quality of Service) support mechanisms are required for network traffic management in wireless networks.

OFDM based wireless LAN traffic may exhibit singular properties related to multifractal characteristics on small time scale due to the IEEE 802.11 MAC protocol mechanisms. Fixed broadband access systems, e.g. IEEE 802.16a, may also present such singularities.

The admission control in wireless communication systems is closely related to the QoS support mechanisms. For this reason, admission control has been regarded as an important scheme in stipulating the correct amount of traffic allowed to be inserted into networks [10], [11].

In this work, we argue that multifractal analysis can enhance the performance of OFDM/TDMA based wireless networks. Thus, by assuming a multifractal traffic model, we propose an expression for calculating the byte loss probability of wireless traffic flows that can be applied to evaluate the queueing performance in terms of byte loss rate of an OFDM/TDMA based wireless system, which was previously presented, by us, in [12]. Afterwards, based on the loss probability expression, we present, as a novelty of this paper, an admission control scheme that decides whether or not to allow new users to the OFDM/TDMA based wireless system.

II. MULTIFRACTAL ANALYSIS

A. Multifractal network traffic

A wireless network traffic flow may present strong dependence among its samples with incidence of bursts at various scales and different scaling laws as found on wired network traffic flows [4],[5].

Multifractal processes are defined by a scaling law for the statistical moments of the processes’ increments over finite time intervals. Now, let us formally define the concept of a multifractal process.

Definition 1: A stochastic process \(X(t)\) is called multifractal if it satisfies

\[
E(|X(t)|^q) = c(q)\tau(q)+1
\]

for \(t \in T\) and \(q \in Q\), where \(T\) and \(Q\) are intervals on the real line, and \(\tau(q)\) and \(c(q)\) are functions with domain \(Q\). \(\tau(q)\) is the scaling function and \(c(q)\) is the moment factor of the multifractal process. Furthermore, we assume that \(T\) and \(Q\) have positive lengths, and that \(0 \in T\), \([0,1] \subseteq Q\). If \(\tau(q)\) is linear in \(q\), the process \(X(t)\) is called monofractal; otherwise, it is multifractal [4].

One of the most important multifractal models present in the literature was proposed by Riedi et al., namely the MWM (Multifractal Wavelet Model) [4]. In the MWM, a positive, stationary, long-range dependent signal \(C(t)\) is represented on the wavelet domain.

Let \(C^{(n)}[k]\) be a discrete-time signal that approximates \(C(t)\) at resolution \(2^{-n}\), where \(n\) is the variable that refers to the time scale. Using the Haar wavelet, the discrete process \(C^{(n)}[k]\) takes values that correspond to the integral of \(C(t)\) in the interval \([k2^{-n},(k+1)2^{-n}][\). Such processes can be mathematically described by the following equation:

\[
C^{(n)}[k] = \int_{k2^{-n}}^{(k+1)2^{-n}} C(t)dt = 2^{-n/2}U_{n,k}
\]

For the Haar wavelet, the scaling coefficients \(U_{j,k}\) and the wavelet transform coefficients \(W_{j,k}\) can be recursively
computed and the simple constraint \( |W_{j,k}| \leq U_{j,k} \) guarantees that the created process \( U_{j,k} \) is positive [4].

In the MWM structure, we can relate the shift \( k_j \) of a scaling coefficient to the shift of one of its two direct descendents \( k_{j+1} \) via \( k_{j+1} = 2k_j + k' \), with \( k_j = 0 \) corresponding to the left descendant and \( k' = 1 \) the right descendant. Fig. 1 illustrates the wavelet domain structure of the MWM. Thus, we can write the MWM scaling and wavelet coefficients as:

\[
U_{j,k} = 2^{-j/2}U_{0,0}\prod_{i=0}^{j-1}[1 + (-1)^k A_{i,k}]
\]

\[
W_{j,k} = 2^{-j/2}A_{j,k}U_{0,0}\prod_{i=0}^{j-1}[1 + (-1)^k A_{i,k}]
\]

The moments of \( C^{(n)}[k] \) can be readily calculated from (5) via [4]:

\[
E[C^{(n)}[k]^p] = E[U_{0,0}^p]\prod_{j=0}^{n-1}\left[1 + A_j \right]^p
\]

\[\text{(6)}\]

B. Scaling function

A multifractal process as defined by equation (1) implies stationary condition for its increments [13]. Thus, the relation given in equation (7), presented below, can be verified for the moments of the increments processes, that is:

\[
E \left[ Z^{(m)}[\tau] \right] = c(q)(\Delta t)^{\tau(q)}
\]

where \( Z^{(m)} \) denotes the increment process of time sample \( \Delta t \), \( q \geq 0 \) and \( \tau(q) = \tau(q) + 1 \). Then, choosing \( \Delta t \) as a time unit, we can rewrite (7) as:

\[
\log E \left[ Z^{(m)}[\tau] \right] = \tau(q)\log m + \log c(q)
\]

since the equation (8) also holds for the aggregated level \( m = 1, 2, \ldots \).

If the sequence of increments \( Z^m \) has scaling property, then the plot of absolute moments \( E\left| Z^{(m)}[\tau] \right| \) versus \( m \) on a log-log plot should be a straight line due to equation (8). The slope of the straight line provides the estimate of \( \tau_0(q) \) and the interception point is the value for \( \log c(q) \).

Once the values of \( \tau_0(q) \) are estimated for different values of \( q \), a curve of \( \tau_0(q) \) or \( \tau(q) \), in function of \( q \) can be built. If the scaling function \( \tau(q) \) versus \( q \) has a linear behavior, it is said that this is an indication of a monofractal behavior of the process, otherwise it is said there is an indication that the process is multifractal.

The scaling function plots for a self-similar traffic trace and for two wireless traffic traces are shown in Fig. 2 [12], where we can observe that the self-similar trace presents a linear relationship between \( \tau(q) \) and \( q \) which is fully consistent to the monofractal scaling behaviour. In contrast, the \( \tau(q) \) function versus \( q \) for the wireless traffic traces, such as the USCtrace01, collected from the USC_06spring_trace packet from USC (University of Southern California), available at [14] and the ISF_wifi dog traffic trace available at [16], show evidences of a nonlinear behavior, suggesting a multifractal structure.

Fig. 2: The scaling function plot for a monofractal traffic trace (dec-pkt-2) [15] and for the wireless traffic traces USCtrace01 [14] and ISF_wifidog [16].

III. THE OFDM/TDMA WIRELESS SYSTEM

We intend to evaluate the efficiency of the proposed loss probability estimation approach applied to an OFDM/TDMA based wireless system. To this end, we consider an OFDM transmission scheme similar to the scenario presented in [17], with \( N \) users and \( M \) traffic channels (i.e., subcarriers) as represented by Fig. 3. The packet arrivals are assumed to be multifractal processes, since we have verified in the previous section that wireless traffic traces can present those multifractal characteristics.
In the considered wireless system, data traffic for each user is buffered into a separate queue and the buffer size is finite. We consider a scenario with characteristics of TDMA-based multiple access with round-robin scheduling. We also assume that the channel state information (i.e., signal-to-noise ratio (SNR)) is available at the transmitter system, and the total transmission bandwidth is \( B \). Then, each subcarrier has a bandwidth of \( \Delta f = \frac{B}{M} \) Hz.

By using adaptive modulation and coding (AMC), the maximum number of bits per symbol (per Hz), denoted by \( c_{m,n}(t) \) that subcarrier \( m \) for user \( n \) can transmit per time unit during time slot \( t \) can be expressed as a function of SNR and target bit error rate (BER). Although, there are several approximations for this function (e.g., [18]), all of them are upper bounded by the following capacity expression [17]:

\[
C_{m,n}(t) = \log_2 \left( 1 + \frac{-1.5}{\ln(5P_{\text{ber}})} \gamma_{m,n}(t) \right)
\]

where \( \gamma_{m,n}(t) \) is the instantaneous SNR at time slot \( t \) for subcarriers \( m \) corresponding to user \( n \) and \( P_{\text{ber}} \) is the target bit error rate (BER).

**IV. LOSS PROBABILITY ESTIMATION**

The workload process \( W(t) \) is the total amount of work stored in the buffer in the time interval \([0, t]\) [13], i.e.,

\[
W(t) = A(t) - ct
\]

where \( A(t) \) is the accumulated amount of work that arrives to the queue model and the service rate is \( c \).

In order to estimate loss probability in a single server link, we focus on the current buffer queueing length, denoted by \( Q \). This is the queue length in the equilibrium state (steady-state) of the queue when the system has been running for a long time and the initial queue length has no influence, using the Lindley’s equation [19], we have:

\[
Q = \sup_{t \geq 0} W(t)
\]

where \( W(0) \) is assumed to be \( 0 \).

As we are considering a finite buffer size \( b \), when the queue size increases in a way to cause overflow of the buffer size, data loss occurs. Mathematically, we have:

\[
P(T) = \sum_{t=0}^{T} \max(Q(t) - c, 0)
\]

where \( Q(0) \) is also assumed to be \( 0 \), \( P \) is the amount of loss and \( T \) is the considered time period. Then, we state that the estimate of the loss probability measure \( P(Q > b) \) corresponds to the relation \( P(T) / A(T) \).

Next, we refer to \( K_r \), as representing the data (traffic intensity) at time scale \( r \). Set \( K_0 = 0 \). For dyadic time scales \( r = 2^m \), the amount of traffic is also related to \( K_{2^m} \), where \( m \in \{0, \ldots, n\} \) and \( n \) is the number of scales considered.

Moreover, tree based wavelet domain models such as the MWM provide explicit and simple formulas of \( K_r \) for dyadic time scales (i.e., \( r = 2^m \)). For such processes the following approximation is valid [3]:

\[
P(Q < b) = \prod_{i=0}^{n} P(K_{2^i} < b + c2^i)
\]

This approach is known as MSQ (Multiscale Queueing), which provides a queueing formula for tree-based wavelet domain models. Another approach, similar to MSQ, is called CDTSQ (Critical Dyadic Time-Scale Queue) which takes into account only the queue length distributions at dyadic time scales [3].

Now, we can enunciate the following proposition regarding data loss as defined in (14) for multifractal traffic [12].

**Proposition 1.** Let \( X(t) \) be a multifractal process whose multipliers \( A_j \) of the corresponding wavelet domain multifractal model possess a symmetric probability distribution, \( b \) is the buffer size and \( c \) is the server capacity. The loss probability for this server is given by:

\[
P(Q > b) = 1 - \prod_{i=0}^{n} \left( 1 - \frac{1}{1 + \sum_{j=0}^{n-1} A_j 2^j} \right)
\]

**V. EVALUATION OF THE PROPOSED LOSS PROBABILITY ESTIMATION APPROACH**

In this section, we apply the proposed loss probability estimation in the OFDM/TDMA wireless system depicted in Fig. 3.

The configuration of the simulation parameters was set as follows. For the specified OFDM/TDMA system, a scenario with 128 subcarriers (i.e., \( M = 128 \)) and total bandwidth \( B \) of 1.920 MHz was considered. The length of a time slot is set to be 10 ms and the packet size is assumed to be 256 bits.

The bandwidth of each subcarrier \( \Delta f \) is 15 kHz. In order to capture the effect of frequency selective fading, the average SNR for each subcarrier is chosen from a Gaussian distribution with mean 15 dB, as has been done in [17]. The targeted BER (Bit Error Rate) was set to be \( 10^{-4} \).

Fig. 4 and Fig. 5 show the loss probability in terms of buffer size. The results were collected from simulations of the considered scenario. We show in this paper the loss probability performance for different number of users in the system. The number of queues and the wireless network users related to Fig. 4 are different to those of the Fig. 5. More precisely, in Fig. 4 the number of users was set to be 10 (i.e, \( N = 10 \)) and represents the loss probability considering as input traffic the USCtrace01[14]. Fig. 5 represents the loss probability for another wireless traffic trace collected from the USC06_spring packet trace, named USCtrace02[14], as input traffic where \( N \) was set to be 5.
It can be noticed that the proposed expression provides more precise values (closer to simulation results) than the other considered methods. We also observed that the other methods present results close to those obtained by simulation only for a small number of buffer sizes. By another hand, the proposed method presented a good performance for a wide range of buffer sizes.

Fig. 4. Comparison of the loss probability obtained with the proposed loss probability expression to other methods for the USCtrace01 \((N=10)\) [14].

Fig. 5. Comparison of the loss probability obtained with the proposed loss probability expression to other methods for the USCtrace02 \((N=5)\) [14].

VI. ADMISSIBLE NUMBER OF USERS BASED ON LOSS PROBABILITY ESTIMATION

In this section, we evaluate the admissible number of wireless traffic flows based on the loss probability estimation given by expression (16). We aim at verifying the impact on the number of admissible flows when the proposed loss probability expression is used for admission control in the OFDM/TDMA system model (Fig. 3). Moreover, we compare the performance of our admission control approach to other methods.

The task of the proposed admission control approach is seemingly simple and can be formulated as follows: suppose that there are \(N\) connections in the OFDM/TDMA system, with queue lengths \(Q_i\) exceeding the buffer size \(b_i\) for each connection \(i\) of the \(N\) connections. When the mean loss probability of the \(N\) connections of the OFDM/TDMA system exceeds the desired loss probability upper bound value denoted by \(\varepsilon\), new connections are not allowed. Mathematically, the admission of a new user must obey the following equation:

\[
\frac{\sum_{i=1}^{N} P(Q_i > b_i)}{N < \varepsilon}
\]

(17)

Fig. 6 and Fig. 7 show the loss probability in terms of the number of users in the OFDM/TDMA system. The results were collected from simulations of the considered scenario.

The simulations were carried out using two different wireless traffic traces, named USCtrace01 and USCtrace02, collected from the USC_06spring_trace packet from USC (University of Southern California), available at [14]. The simulations were also evaluated using two different buffer sizes.

Fig. 6 represents the loss probability considering as input traffic the USCtrace01, where the buffer size was set to be \(0.8\times10^7\) bytes, and Fig. 7 represents the loss probability for the USCtrace02 as input traffic, where the buffer size was set to be \(2\times10^7\) bytes.

Through Figures 6 and 7, it can be noticed that the proposed admission control scheme based on the loss probability expression (16) provides more precise values (closer to simulation results) than the other considered methods. That is, the admission control can decide whether to accept or not a traffic flow based on some of its characteristics (behavior of the cascade multipliers across scales). Assuming that the characteristics of the traffic will be approximately the same, the admission control intelligence can predict the performance of the wireless system if the traffic flow was accepted. Also, the proposed expression follows the loss probability curve estimated by simulation for a wide range of number of users.

Table I and Table II show how many users are accepted in the OFDM/TDMA system for different loss probability upper
bound values ($\varepsilon$) for the USCTrace01 e USCTrace02 traffic traces, respectively. We can notice that the proposed scheme present better results than, or comparables to, the MSQ and CTDTSQ methods. That is, a larger number of users is admitted to the OFDM/TDMA system for the same target loss probability.

### TABLE I. NUMBER OF USERS ACCEPTED IN THE OFDM/TDMA SYSTEM RELATED TO $\varepsilon$ (USCTRACE01)

<table>
<thead>
<tr>
<th>$\varepsilon$</th>
<th>Simulation</th>
<th>Proposed</th>
<th>MSQ</th>
<th>CTDTSQ</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1.0\times10^{-2}$</td>
<td>2</td>
<td>1</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>$2.5\times10^{-2}$</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>$5.0\times10^{-2}$</td>
<td>7</td>
<td>7</td>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>$7.5\times10^{-2}$</td>
<td>9</td>
<td>10</td>
<td>4</td>
<td>7</td>
</tr>
<tr>
<td>$1.0\times10^{-1}$</td>
<td>10</td>
<td>10</td>
<td>5</td>
<td>8</td>
</tr>
</tbody>
</table>

### TABLE II. NUMBER OF USERS ACCEPTED IN THE OFDM/TDMA SYSTEM RELATED TO $\varepsilon$ (USCTRACE02)

<table>
<thead>
<tr>
<th>$\varepsilon$</th>
<th>Simulation</th>
<th>Proposed</th>
<th>MSQ</th>
<th>CTDTSQ</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1.0\times10^{-2}$</td>
<td>0</td>
<td>1</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>$2.5\times10^{-2}$</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>$5.0\times10^{-2}$</td>
<td>6</td>
<td>6</td>
<td>5</td>
<td>7</td>
</tr>
<tr>
<td>$7.5\times10^{-2}$</td>
<td>8</td>
<td>10</td>
<td>6</td>
<td>8</td>
</tr>
<tr>
<td>$1.0\times10^{-1}$</td>
<td>10</td>
<td>10</td>
<td>6</td>
<td>10</td>
</tr>
</tbody>
</table>

### VII. CONCLUSION

The characteristics of traffic flows especially in wireless networks, as long-range dependence and bursts at multiple scales make traffic modeling a difficult and challenging task. In this paper, we first showed that the multifractal analysis is suitable to characterize wireless network traffic.

In order to evaluate loss probability in wireless networks, we considered a simplified wireless scenario operating with an OFDM/TDMA scheme, where a round robin scheduling controls the capacity sharing and data transmission. By considering multifractal characteristics, we obtained an analytical expression to determine the loss probability for wireless traffic traces applied to a wireless scenario. Based on the loss probability expression we proposed a simple admission control scheme which is applied to the OFDM/TDMA system model. Therefore, in the considered scenario we verified the influence on the number of admissible flows when the proposed loss probability expression is used as a criteria for admission control.

Comparisons to other loss probability approaches showed that our proposal of admission control provided more precise results regarding the considered real wireless traffic traces.

The experimental results considering an OFDM/TDMA wireless system showed that the loss probability estimation given by expression (16) is adequate to be used as basis for a simple admission control scheme, since we verified the proposed scheme present better results than the MSQ and CTDTSQ methods.

### APPENDIX I: PROOF OF PROPOSITION 1

Let $X$ be a non-negative random variable and $a$ a possible value that $X$ can assume. Then, Markov’s Inequality guarantees that [20]:

$$P[X < a] > 1 - \frac{E[X]}{a}$$  

(18)

In addition, the inequality (18) can be rewritten for a dyadic process $K$, as:

$$P[K_i < b + c2^i] > 1 - \frac{E[K_i]}{b + c2^i}$$  

(19)

This result can be reintroduced into equation (15), resulting in the following expression:

$$P[Q < b] \approx \prod_{i=0}^{n} \left(1 - \frac{E[U_{i,0}]}{b + c2^i}\right)$$  

(20)

By substituting equation (6), into (20), we obtain the desired loss probability expression:

$$P[Q > b] \approx 1 - \prod_{i=0}^{n} \left[1 - \frac{E[U_{i,0}]}{b + c2^i}\right]$$  

(21)

### REFERENCES


