Hybrid Local Search Polynomial-Expanded Linear Multiuser Detector

Reinaldo Götz and Taufik Abrão

***Abstract*— In order to reduce computational complexity inherent to cross-correlation matrix inversion in DS/CDMA system, this work proposes a hybrid multiuser detector based on polynomial expansion (PE-MuD) followed by a low complexity local search procedure, aiming at obtaining a near-optimum multiuser bit-error-rate (BER) performance, but with an amount of computational processing saving time. The proposed hybrid PE-MuD receiver topology is analyzed under realistic wireless mobile channels, as well as useful system operation scenarios. Simulations results have indicated an improvement in performance-complexity trade-off regarding the classical linear multiuser detectors (MuD) performance, particularly, the mean square error minimization-based detector (MMSE).**

***Keywords*— suboptimum search algorithms, polynomial expanded multiuser detection, Gerschgorin circles, DS/CDMA.**

I. INTRODUCTION

The total use of the transmission channel capacity, regardless of the channel adopted, depends on the features of the detector utilized, which prevent the effects generated by the multiple access through non orthogonal code division, mainly, in the effectiveness of the receptor to mitigate the effects of multiple access interference (MAI), as well as to deal with the near-far ratio (NFR). The optimal solution for the multiuser detection problem lies in the employment of maximum likelihood (ML) detector, presented in [1]. However, ML detector complexity is impractical in almost scenarios of interest. Hence, linear near-optimal MuDs, such as the Decorrelator and MMSE were proposed in [2], [3]. Basically, these detectors utilize the inverse cross-correlation matrix of signature waveforms of the active users in the system ($\mathbf{R}^{-1}$) to decouple the desired user's signal.

Aiming at more efficient linear detectors implementation, a multiple stage detection scheme, which obtains the inverse cross-correlation matrix through polynomial expansion in $\mathbf{R}$, has been presented in [4]. In this study, the cross-correlation matrix inversion is approximated via Neumann iterative series expansion, with its coefficients estimated by the Gerschgorin circles method [5].

The local search detection method, which sometimes is classified as heuristic, but, in fact is a deterministic one, is an optimization method that consists in the search of solutions in a previously established neighborhood [6]. The main advantage of this method lies on its reduced complexity. A structure formed by the polynomial detector as the first stage followed by a local search algorithm that provides a gain in the detector performance has been presented in [7].

This work is divided into six Sections. Besides this introductory section, the system model is established in Section II, in which a review on classic linear single-user (SuD) and multiuser detectors (MuDs) is presented. The polynomial expansion method, used in the inverse cross-correlation matrix approximation, is discussed in Section III. The application of the local search method in the multiuser detection problem is addressed in Section IV. The performance of some of the MuD methods is revealed in Section V, while the work's conclusions are summarized in Section VI.

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II. SYSTEM MODEL

Here, a discrete-time baseband system model is adopted, with transmission through a channel with a single antenna in the transmitter and receptor (SISO – *single-input single-output*) subjected to additive white Gaussian noise (AWGN) and flat Rayleigh fading. The same channel is simultaneously shared by K users, which operate under a synchronous DS/CDMA system with binary phase shift keying modulation (BPSK). In the transmission, the ith information bit generated by the kth user, at a ratio of $R_{\rm b} = 1/T_{\rm b}$ bits per second is denoted by $b_k[i] \in \{\pm 1\}$, $i = 1, 2, \ldots$. At each i bit interval, $b_k[i]$ is modulated by a spread sequence with pseudo-noise (PN) distribution and the ratio of $R_{\rm c} = 1/T_{\rm c} = N/T_{\rm b} = NR_{\rm b}$ chips per second, represented by the vector

$$\mathbf{s}_k[i] = \left(s_{k,1}[i], s_{k,2}[i], \ldots, s_{k,N}[i]\right)^{\rm T}, \qquad (1)$$

with $s_{k,n}[i] \in \left\{\pm 1/\sqrt{N}\right\}$ and N denoting the system's processing gain; $(\cdot)^{\rm T}$ denotes the matrix transposing operator.

In the base radio station (BRS), the received signal vector is represented by

$$\mathbf{r}[i] = \sum_{k=1}^{K} \mathbf{s}_k[i]\, c_k[i]\, A_k b_k[i] + \mathbf{n}[i], \qquad (2)$$

where A_k is the amplitude of the signal transmitted by the kth user; $\mathbf{n}[i]$ is the complex AWGN vector of mean zero and variance $\sigma_{\rm n}^2 = N_0$, with bilateral power spectral density of AWGN noise given by $N_0/2$ W/Hz.

The term $c_k[i]$ denotes the complex coefficient of the channel inherent to the kth user, at the i bit interval, perfectly known by the receptor. In statistical terms, $c_k[i]$ may be represented by a circularly symmetric complex Gaussian random variable, with mean zero and variance $\sigma_{\rm c}^2$, in the form $\mathcal{CN}\left(0, \sigma_{\rm c}^2\right)$. In the polar form, the channel's complex coefficient is described by:

$$c_k[i] = |c_k[i]|\, e^{j\theta_k[i]}, \qquad (3)$$

where phase $\theta_k[i]$ is uniform over the range $[0, 2\pi)$ and independent of the magnitude $|c_k[i]|$, whose probability density function is given by Rayleigh, $f(r) = \frac{r}{\sigma_{\rm c}^2} e^{-r^2/2\sigma_{\rm c}^2},\ r \geq 0$.

In the notation of matrices, with bold capital letters representing matrices and bold lower case letters representing vectors, and suppressing the term i for the sake of convenience, the Equation (2) may be rewritten as follows:

$$\mathbf{r} = \mathbf{SCAb} + \mathbf{n}, \tag{4}$$

with $\mathbf{A} = \mathrm{diag}\left(A_1,\, A_2, \ldots,\, A_K\right)$ being the amplitude diagonal matrix of the received signals, $\mathbf{S}$ the spread spectrum matrix with dimensions $N \times K$ and $\mathbf{C} = \mathrm{diag}\left(c_1,\, c_2, \ldots,\, c_K\right) = \mathrm{diag}\left(|c_1|,\, |c_2|, \ldots,\, |c_K|\right) \mathrm{diag}\left(e^{j\theta_1},\, e^{j\theta_2}, \ldots,\, e^{j\theta_K}\right) = \mathbf{FP}$ corresponding to the channel complex coefficients matrix, where $\mathbf{F}$ $\mathbf{P}$ are respectively the diagonal matrices of magnitudes and phases of the channel. Vector $\mathbf{b} = \left(b_1,\, b_2, \ldots,\, b_K\right)^{\mathrm{T}}$ contains information bits transmitted by the K users and $\mathbf{n} = \left(n_1,\, n_2, \ldots,\, n_K\right)^{\mathrm{T}}$ is the complex noise vector with the distribution $\mathcal{N}\left(0,\, \sigma_{\mathrm{n}}^2\right)$.

The output signal of the matched filters bank (MFB) is described taking account the channel phases:

$$\begin{aligned} \mathbf{y}_{\mathrm{mfb}} &= \mathbf{P}^*\mathbf{y} = \mathbf{P}^*\mathbf{S}^{\mathrm{T}}\mathbf{r} = \mathbf{S}^{\mathrm{T}}\mathbf{S}|\mathbf{C}|\mathbf{Ab} + \mathbf{P}^*\mathbf{S}^{\mathrm{T}}\mathbf{n} \\ &= \mathbf{RFAb} + \mathbf{z}, \end{aligned} \tag{5}$$

where vector $\mathbf{y} = \left(y_1, y_2, \ldots, y_K\right)^{\mathrm{T}}$ represents the despread baseband-received signal, whose components are given by $y_k = \mathbf{s}_k^{\mathrm{T}}\mathbf{r}$; the cross-correlation matrix of the signature waveforms is obtained via $\mathbf{R} = \mathbf{S}^{\mathrm{T}}\mathbf{S}$; vector $\mathbf{z} = \mathbf{P}^*\mathbf{S}^{\mathrm{T}}\mathbf{n}$ corresponds to the filtered noise with variance $\sigma_{\mathrm{n}}^2\mathbf{R}$; the conjugate operator is denoted by $(\cdot)^*$. Finally, the K users' information bits vector is estimated through:

$$\widehat{\mathbf{b}}_{\mathrm{conv}} = \mathrm{sgn}\left(\Re\left\{\mathbf{y}_{\mathrm{mfb}}\right\}\right), \tag{6}$$

where $\Re\{\cdot\}$ is the real part operator.

A. Optimum Detection

The optimum performance is obtained with the use of the ML detector, which performs the joint information detection of the K users in the system, maximizing the following cost function, which is based on the Euclidean distance between the received signal and the signal reconstructed in the receptor from the information candidate vector, $\underline{\mathbf{b}}$:

$$\Omega\left(\underline{\mathbf{b}}\right) = 2\Re\left\{\mathbf{y}^{\mathrm{T}}\mathbf{C}^{\mathrm{H}}\mathbf{A}\underline{\mathbf{b}}\right\} - \underline{\mathbf{b}}^{\mathrm{T}}\mathbf{CARAC}^{\mathrm{H}}\underline{\mathbf{b}}, \tag{7}$$

where $(\cdot)^{\mathrm{H}}$ is the matricial operator of conjugation and transposition.

The optimum multiuser detection (OMuD) criterion yields the best information bits estimated vector $\widehat{\mathbf{b}}_{\mathrm{opt}}$:

$$\widehat{\mathbf{b}}_{\mathrm{opt}} = \arg\left\{\max_{\underline{\mathbf{b}} \in \mathcal{M}^{\mathcal{P}K}} \left\{\Omega\left(\underline{\mathbf{b}}\right)\right\}\right\}, \tag{8}$$

where $\mathcal{P}$ is the transmitted message length and $\mathcal{M}$ the symbol alphabet dimension. For the binary modulation, i.e. $\mathcal{M} = 2$, the computational complexity of the ML detector is of the order of $\mathcal{O}\left(2^K\right)$.

B. Linear Methods of Multiuser Detection

In [2], the linear methods of detection were discussed, including the Decorrelator detector. This one operates from multiplication of the discrete signals at the matched filters output by the inverse cross-correlation matrix $\mathbf{R}^{-1}$. Considering the coherent reception model, the information bits vector which is estimated after the application of the Decorrelator multiuser filter may be described as follows:

$$\begin{aligned} \widehat{\mathbf{b}}_{\mathrm{dec}} &= \mathrm{sgn}\left(\Re\left\{\mathbf{R}^{-1}\mathbf{y}_{\mathrm{mfb}}\right\}\right) \\ &= \mathrm{sgn}\left(\Re\left\{\mathbf{R}^{-1}\mathbf{RFAb} + \mathbf{R}^{-1}\mathbf{z}\right\}\right) \\ &= \mathrm{sgn}\left(\Re\left\{\mathbf{FAb} + \mathbf{w}\right\}\right). \end{aligned} \tag{9}$$

Thus, from (5), the decision variable for kth user may be individually obtained as

$$\Re\left\{\left(\mathbf{R}^{-1}\mathbf{y}\right)_k e^{-j\theta_k}\right\}, \tag{10}$$

where $(\cdot)_k$ selects the kth element of the argument vector. The Decorrelator detector presents a gain in the performance, in relation to the Conventional detector, although the power associated to the noise term, obtained at the Decorrelator output, is always bigger or equal to the noise term obtained at the Conventional output. Another linear detection method known in literature is the MMSE detector, proposed for CDMA systems in [3]. This method is based on the appropriate choice of a linear transformation vector, $\mathbf{t}_k = \left(t_1,\, t_2, \ldots, t_K\right)^{\mathrm{T}}$, that minimizes the mean square error between the kth user's information bit and the kth linear transformations output, $\mathbf{t}_k\mathbf{y}_{\mathrm{mfb}}$, resulting in:

$$\min_{\mathbf{t}_k} \mathbb{E}\left\{\left(b_k - \mathbf{t}_k\mathbf{y}_{\mathrm{mfb}}\right)^2\right\}. \tag{11}$$

The vector that minimizes (11) involves the covariance of colored noise $\mathbf{z}$ and the estimated amplitude of the users in the receiver, $\mathbf{B} = |\mathbf{C}|\mathbf{A}$. By applying this solution to the joint detection of the K users, the $K \times K$ transformation matrix $\mathbf{T}$ is given by:

$$\mathbf{T} = \left[\mathbf{R} + \sigma_{\mathrm{n}}^2\mathbf{B}^{-2}\right]^{-1}. \tag{12}$$

Therefore, the output vector of the linear MMSE detector follows the decision:

$$\widehat{\mathbf{b}}_{\mathrm{mmse}} = \mathrm{sgn}\left(\Re\left\{\mathbf{T}\mathbf{y}_{\mathrm{mfb}}\right\}\right). \tag{13}$$

III. Polynomial-Expanded Multiuser Detectors

The computational complexity of the linear MuDs, which originates in the operations associated to the cross-correlation matrix inversion, grows with the third order of the matrix size, i.e., $\mathcal{O}\left(K^3\right)$. However, the inverse matrix $\mathbf{R}^{-1}$ can be approximated through the polynomial expansion method, resulting in the polynomial-expanded multiuser detector (PE-MuD), with complexity of $\mathcal{O}\left(K^2\right)$. The resulting information estimated vector is given by:

$$\widehat{\mathbf{b}}_{\mathrm{pe}} = \mathbf{L}_{\mathrm{pe}}\mathbf{y}, \tag{14}$$

with $\mathbf{L}_{\mathrm{pe}}$ being the iterative polynomial expansion for the linear transformation factor $\mathbf{T}$ of a specific linear detector:

$$\mathbf{L}_{\mathrm{pe}} = \sum_{i=0}^{N_{\mathrm{t}}} \alpha_i \widetilde{\mathbf{T}}^i, \tag{15}$$

where N_{t} indicates the number of terms of the polynomial expansion in the PE-MuD detector, $\underset{\sim}{\alpha_i}$ denotes the coefficients for the series convergence rate, and $\widetilde{\mathbf{T}}$ describes the transformation matrix for the linear multiuser detectors Decorrelator or MMSE.

A. Polynomial Expansion via Neumann Series

By using the Neumann series expansion method [5], the inverse cross-correlation matrix $\mathbf{R}^{-1}$ may be approximated:

$$\mathbf{R}^{-1} \approx \alpha \sum_{i=0}^{N_t} \left(\mathbf{I}_K - \alpha \mathbf{R}\right)^i, \quad \|\mathbf{I}_K - \alpha \mathbf{R}\| < 1 \tag{16}$$

where $\mathbf{I}_K$ is a identity matrix of size K, and the associated residual error is:

$$\boldsymbol{\varepsilon}_{\mathbf{R}_{\rm inv}} = \alpha \sum_{i=N_t+1}^{\infty} \left(\mathbf{I}_K - \alpha \mathbf{R}\right)^i. \tag{17}$$

In turn, the transformation factor of the linear MMSE detector, defined by $\mathbf{T} = \left(\mathbf{R} + \sigma_{\rm n}^2 \mathbf{B}^{-2}\right)^{-1}$, may be approximated in:

$$\left(\mathbf{R} + \sigma_{\rm n}^2 \mathbf{B}^{-2}\right)^{-1} \approx \alpha \sum_{i=0}^{N_t} \left[\mathbf{I}_K - \alpha \left(\mathbf{R} + \sigma_{\rm n}^2 \mathbf{B}^{-2}\right)\right]^i. \tag{18}$$

In the Equation (16), the convergence factor of the Neumann series is equal to the spectral radius[1] of the matricial operator, $\rho\left(\mathbf{I}_K - \alpha \mathbf{R}\right)$. Therefore, the series converges if the spectral radius' value is less than one [8]. As a consequence, the series converges with any scalar α which satisfies

$$0 < \alpha < \frac{2}{|\lambda_{\max}|}. \tag{19}$$

For the case of linear MMSE detector, the convergence factor is defined here with the premise that the users' amplitude estimated matrix is $\mathbf{B} = \mathbf{I}_K$, as follows:

$$\left\|\mathbf{I}_K - \alpha \left(\mathbf{R} + \sigma_{\rm n}^2 \mathbf{I}_K\right)\right\| < 1.$$

Therefore, the parameter α that allows the convergence of the MMSE detector' approximation is found in the interval

$$0 < \alpha < \frac{2}{|\lambda_{\max} + \sigma_{\rm n}^2|}. \tag{20}$$

B. Optimum Value of the Parameter α

Since the convergence factor of an iterative method can be associated with the matricial operator' radius, the convergence ratio of this method is related to the dimension of this radius [8]. For the matricial operator which approximates the Decorrelator, the scalar α that optimizes the spectral radius is achieved by the equation:

$$\alpha_{\rm opt}^{\rm dec} = \frac{2}{\lambda_{\min} + \lambda_{\max}}. \tag{21}$$

For the case of linear MMSE detector, the optimum value of α is given by:

$$\alpha_{\rm opt}^{\rm mmse} = \frac{2}{\lambda_{\min} + \lambda_{\max} + 2\sigma_{\rm n}^2}. \tag{22}$$

[1]Spectral radius of a matrix corresponds to the absolute value of its greater eigenvalue.

C. Gerschgorin Circles Theorem

The optimum value of α can be estimated, by using the Gerschgorin circles theorem [5]. According to this theorem, any eigenvalue of a matrix $\mathbf{R}$, with elements $r_{i,j}$, $\forall i,j$, is situated in one of the complex plan' circles that are centered in $r_{i,i}$, with radius $\sum_{i,j\neq i} |r_{i,j}|$, i.e.,

$$|\lambda_i - r_{i,i}| \leq \sum_{i,j\neq i} |r_{i,j}|. \tag{23}$$

Thus, through a simple calculation, by using the elements of $\mathbf{R}$, the approximated values of $\lambda_{\min}$ and $\lambda_{\max}$, which are denoted by $\widehat{\lambda}_{\min}$ and $\widehat{\lambda}_{\max}$, respectively, can be achieved by:

$$\widehat{\lambda}_{\min} \approx \min\left\{ r_{i,i} + \sum_{i,j\neq i} |r_{i,j}| \right\}, \quad \forall i, \tag{24}$$

$$\widehat{\lambda}_{\max} \approx \max\left\{ r_{i,i} + \sum_{i,j\neq i} |r_{i,j}| \right\}, \quad \forall i. \tag{25}$$

The Gesrchgorin circles (GC) theorem allows a considerable reduction in the complexity of the minimum and maximum eigenvalues' calculation, being, therefore, adopted in this work.

IV. Local Search Methods Applied to the Multiuser Detection

Local search methods propitiate the attainment of near-optimum solutions from searches guided in subspaces of the optimization problem' dimension. The deterministic algorithm 1-opt LS (*one-optimum local search*) performs a search for the vector that maximizes the cost function, selecting candidate vectors situated in the unitary Hamming distance[2] from the output vector of MFB. The pseudo-code for the local search algorithm 1-opt LS can be seen in [9]. In the following, an adaptation for the 1-opt LS algorithm is proposed and a new algorithm is formed.

A. Local Search Algorithm 1-adapt LS

The quantity of calculations of the cost functions during the search for the best candidate vector can be limited by using a given threshold. Chase establishes a threshold criterion based on channel measurement informations, by selecting a fixed number of the lowest confidence bits to be changed [10]. Differently of Chase search stop criterion, herein for the proposed 1-opt LS algorithm, a dynamic threshold is used in order to create adaptation and reduce complexity. This new algorithm, namely *one-adaptive local search* (1-adapt LS), classifies the received signals in order of increasing amplitude. Then, candidate vectors with unitary Hamming distance are generated, following the ordering of the signals (from the weakest to the strongest), and their respective cost functions are evaluated. In case of the cost function value is not increase following a pre-established quantity of consecutive evaluations, denoted by κ, the search process is interrupted and a new search is initiated. The pseudo-code for the algorithm 1-adapt LS is described in the Algorithm 1.

[2]Hamming distance between two vectors, e.g., $\mathbf{b}_1$ and $\mathbf{b}_2$, is defined by $d_{\rm H}\left(\mathbf{b}_1, \mathbf{b}_2\right) = \|\mathbf{b}_1 - \mathbf{b}_2\|$, which corresponds to the amount of elements that differ between the vectors.

B. Hybrid 1opt-LS-MuD and 1adapt-LS-MuD Detectors

The detection structure presented in [7] has been reproduced herein, by deploying in the first stage, the polynomial MMSE detector with α estimated via the Gerschgorin circles method, and in the second stage, the 1-opt LS algorithm. The performance results achieved by this Hybrid 1opt-LS-MuD detector are presented in Subsection V-A.

It is worth noting that this work introduces for the first time a multiuser detector constituted by the polynomial MMSE detector followed by a new local search algorithm 1-adapt LS, namely the Hybrid 1adapt-LS-MuD. In subsection V-A a performance comparison including both hybrid sub-optimal multiuser detectors have been carried out.

Algorithm 1 1-adapt LS

Input: $\widehat{\mathbf{b}}_{\text{conv}}$, N_{it}, κ; **Output:** $\widehat{\mathbf{b}}$
begin
1. Initialize the local search: $t = 1$, $l = 0$;
  $\mathbf{b}_{\text{best}}[1] = \widehat{\mathbf{b}}_{\text{conv}}$;
  $g_{\text{best}}[1] = \Omega(\mathbf{b}_{\text{best}}[1])$;
  $g_{\text{ref}}[1] = g_{\text{best}}[1]$;
2. for $t = 1, \dots, N_{\text{it}}$,
    while $l < \kappa$,
      a. Classify signals (increasing amplitude order), given:
        $A_i[t]$, $i = 1, \dots, K$, com $A_i[t] \leq A_{i+1}[t]$;
      b. Generate candidate-vectors with unitary Hamming distance denoted by $\underline{\mathbf{b}}_i[t]$, $i = 1, \dots, K$;
      c. Calculate $g_i = \Omega(\underline{\mathbf{b}}_i[t])$;
      if $g_i[t] > g_{\text{best}}[t]$,
        $g_{\text{best}}[t] \leftarrow g_i[t]$;
        $l = 0$;
      else
        $l = l + 1$;
      end
    $\mathbf{b}_{\text{best}}[t+1] \leftarrow \underline{\mathbf{b}}_{\text{i}}[t]$;
    $g_{\text{ref}}[t+1] \leftarrow g_{\text{best}}[t]$;
    end
  end
  if $g_{\text{ref}}[t+1] = g_{\text{ref}}[t]$, go to step 3
  end
3. $\widehat{\mathbf{b}} = \mathbf{b}_{\text{best}}$;
end

V. PERFORM ANALYSIS

In this section, the performances of the sub-optimal MuDs are evaluated, by means of Monte Carlo simulation (MCS) method. The flat Rayleigh fading channels, which magnitude and phase coefficients are perfectly estimated at the receptor side, have been adopted. The average signal-to-noise ratio (SNR) considered in simulations ranges from 0 to 40 dB. In all numerical results presented in this section, the average SNR, denoted by SNR_{avg}, is deployed in the context of the near-far effect, i.e., there are two interfering group of users with near-far ratio NFR $= P_{\text{interf}}$ (dB) $- P_{\text{interest}}$ (dB) $= +5$ dB ($K/3$ users), and $K/3$ users with NFR $= -5$ dB. Hence, the average SNR and bit-error-rate (BER_{avg}) presented in this section is taking over the interest users group only ($K/3$ users). The adopted processing gain of the DS/CDMA system was $N = 36$. Furthermore, the number of terms in polynomial expansion is limited to $N_{\text{t}} = [1;\ 7]$ terms, while the number of local search algorithm' iterations is limited to $N_{\text{it}} = [0;\ 10]$ iterations. Besides, in the algorithm 1-adapt LS, a good performance-complexity trade-off was achieved with $\kappa = \lceil 0.6 \cdot K \rceil$.

A. 1opt-LS-MuD and 1adapt-LS-MuD

Fig. 1 and 2 show the average BER and the average quantity of cost function calculations by iteration (ζ_{avg}), respectively, for the 1opt-LS-MuD and 1adapt-LS-MuD, as a function of an increasing number of users (system loading robustness). Both figures were obtained from the same MCS setup, considering the same point of system operation and $\text{SNR}_{\text{avg}} = 14$ dB. In this scenario, the quantity of active users in the system ranges from $K = [9;\ 36]$ users, i.e., system loading lies on the range $\mathcal{L} = 100 \cdot K/N = [25\%;\ 100\%]$.

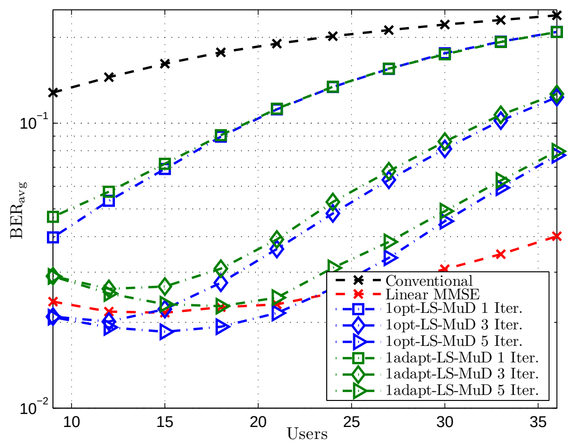

Fig. 1. Comparison between 1opt-LS-MuD and 1adapt-LS-MuD detectors, in the flat Rayleigh channel and $\text{SNR}_{\text{avg}} = 14$ dB.

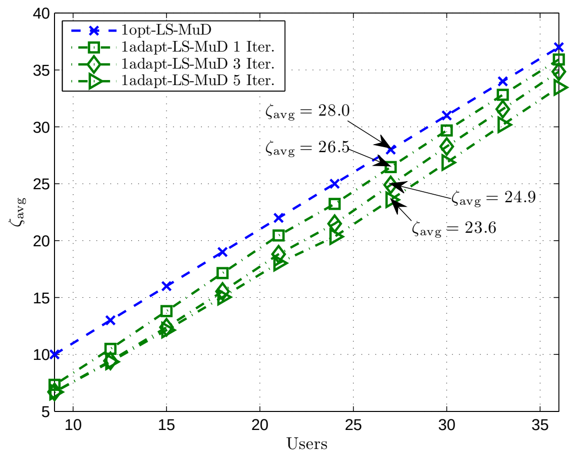

Fig. 2. Average quantity of cost function calculations necessary in the 1opt-LS-MuD and 1adapt-LS-MuD; $\text{SNR}_{\text{avg}} = 14$ dB.

Except in the region of low system loading $\mathcal{L} \leq 33\%$, the performance of 1opt-LS-MuD and 1adapt-LS-MuD detectors with 1 iteration are practically identical. Taking into account $K = 30$ users in the system ($\mathcal{L} = 83,3\%$), the BER performances of the evaluated detectors are very close. Nevertheless, the complexity of the second detector is smaller, according to Fig. 2. By loading the system in 75%, the value of ζ_{avg} accomplished for 1adapt-LS-MuD detector with three iterations is 11% smaller in relation to 1opt-LS-MuD detector, although with a relative increase of 7.1% in the BER_{avg} of the detector with lower complexity. However, as one can conclude from Fig. 1, nether the proposed Hybrid local search PE multiuser detectors (1opt-LS-MuD and 1adapt-LS-MuD) nor the Linear MMSE are completely robust against the system loading (MAI) increasing.

B. Hybrid 1adapt-LS-MuD Detector

As shown in Fig. 3, the Hybrid 1adapt-LS-MuD detector with 1 term in the polynomial expansion does not converge to the ML detector; while with 5 or 7 terms and $\kappa = \lceil 0.6 \cdot K \rceil$, the convergence is guaranteed within 2 iterations. On the other hand, with $N_{\rm t} = 3$ terms, the performance of the Hybrid 1adapt-LS-MuD detector is nearly optimum within $N_{\rm it} = 3$ iterations.

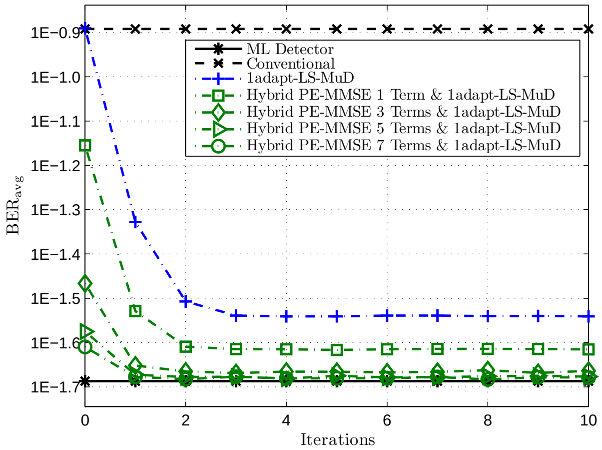

Fig. 3. Convergence of the Hybrid 1adapt-LS-MuD detector in the flat Rayleigh channel and $\text{SNR}_{\text{avg}} = 14$ dB; $K = 9$ users.

Fig. 4 shows the near-far robustness of the Hybrid 1adapt-LS-MuD detector, in comparison with the polynomial and the linear MMSE detectors. In this simulation, $K_{\text{interf}} = 4$ and $K_{\text{interest}} = 4$ users have been considered. One can see that both Linear MMSE and the proposed Hybrid 1-adpat-LS MuDs are extremely robust against the NFR effect.

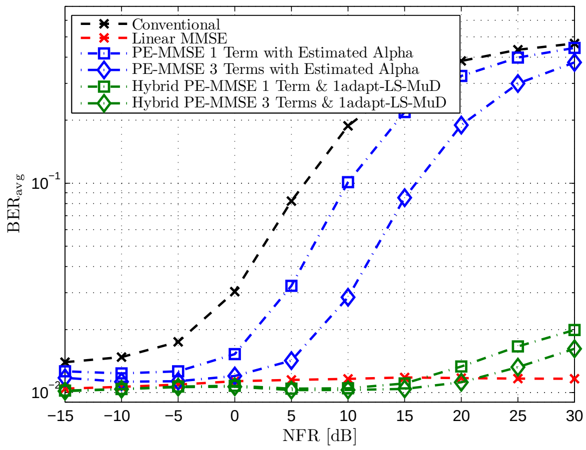

Fig. 4. Near-far effect resistance of the Hybrid 1adapt-LS-MuD detector, in the flat Rayleigh channel and $\text{SNR}_{\text{avg}} = 14$ dB.

Fig. 5 shows that the Hybrid 1adapt-LS-MuD detector with $N_{\rm t} = 1$ term keeps its performance close to that achieved by the linear MMSE-MuD up to SNR $= 12$ dB, i.e., in the low-medium SNR region. However, with $N_{\rm t} = 3$ terms, this performance is extended up to SNR $= 32$ dB. These results represent an excellent performance-complexity trade-off for the proposed hybrid adaptive local search multiuser detector.

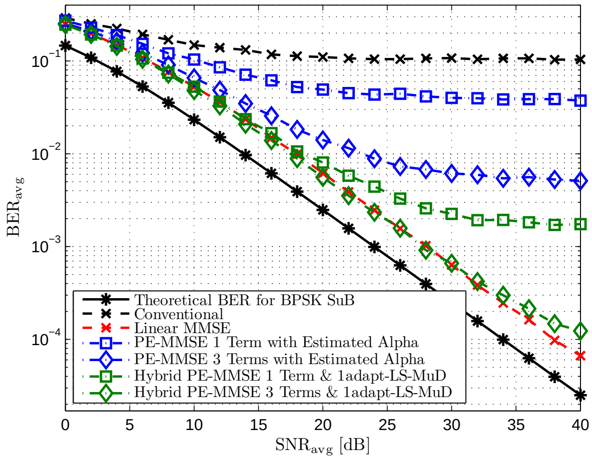

Fig. 5. Hybrid 1adapt-LS-MuD detector performance, in the flat Rayleigh channel; Algorithm 1 with $N_{\rm it} = 3$; $K = 9$ users.

VI. CONCLUSIONS

The proposed local search algorithm 1-adapt LS promotes a remarkable gain in the DS/CDMA system performance equipped with polynomial expansion-based hybrid multiuser detectors. When associated to low-complexity PE-MuD detectors, it provides reliability to the detection process, without an excessive increasing in its implementation cost, been able to offer a good performance-complexity trade-offs.

Simulation results have shown that the proposed 1-adapt LS is able to provide a considerable level of robustness against the near-far effect when combined to the PE-MuD. Furthermore, this hybrid detector achieves fast convergence by using only three terms in the polynomial expansion, with a remarkable trade-off between near-optimum performance and reduced complexity, specially when the detector operates in scenarios with medium or low system loadings and moderate or low NFR.

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