Improving the Performance of Adaptive Modulation Systems by the Modulation Diversity Technique

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Abstract—This paper proposes a combination of the adaptive modulation with the modulation diversity technique, which associates a suitable constellation rotation angle $\theta$ with the independent interleaving of the symbol components. Since the optimal rotation angle depends on the chosen modulation order and fading severity degree, the constellation rotation angle is dynamically changed according to the channel parameters. A new approximate expression for the pairwise error probability (PEP) is derived to allow the dynamic evaluation of the optimal value of $\theta$. A performance analysis considering a system with parameters similar to the UMTS/HSDPA protocol is performed. The evaluation of the proposed system has shown that significant gains, in terms of the average system bit error rate, are achieved for high signal-to-noise ratio values.

Keywords—Adaptive modulation, Modulation diversity, Performance evaluation.

I. INTRODUCTION

Adaptive modulation was proposed to improve the spectral efficiency of a radio link and to ensure a maximum system BER (Bit Error Rate) that satisfies the applications requirements [1]. In this scheme, the system data rate is dynamically adapted by changing the signal constellation order $M$ according to the fading amplitude fluctuations. The adaptation is commonly based on the received channel SNR (Signal-to-Noise Ratio), periodically estimated (or predicted) and returned to the transmitter by the receiver.

The adaptive modulation technique has been widely adopted in different access networks technologies, such as some IEEE 802.11 network technologies, WiMAX (IEEE 802.16e) and UMTS/HSDPA (Universal Mobile Telecommunications System/High-Speed Downlink Packet Access) protocol. However, the performance of adaptive modulation systems can be improved if diversity techniques are employed. Diversity techniques consist, basically, of providing replicas of the transmitted signals to the receiver. Typical examples of diversity techniques are: time, frequency and spatial diversity.

Another proposed diversity technique is based on the combination of a suitable constellation rotation angle (named $\theta$), which is an important design criterion in this scheme, with the independent interleaving of the symbol components before transmission [2]. The optimal rotation angle depends on the constellation order ($M$) and fading intensity degree [3]. This technique is called modulation diversity [4], but it is also known as constellation rotation [5] and signal space diversity [6].

The dynamic characteristics of the wireless communication channels imply that the optimal performance of the modulation diversity technique can only be achieved if the rotation angle is dynamically changed according to the communication channel parameters. In this context, this paper presents the design and evaluation of an adaptive modulation scheme which uses the modulation diversity technique. The estimated channel parameters, channel SNR and fading intensity, are used to adapt the constellation order and the rotation angle.

In order to dynamically determine the optimal value of $\theta$, a new approximate expression for the pairwise error probability (PEP) is derived, which is simple, sufficiently accurate and adequate for the evaluation of the $\theta$ angle. The system evaluation was performed considering a slow Nakagami-$m$ flat fading channel.

The remaining sections are organized as follows. Section II presents the system and channel models, as well as details about the architecture of the proposed solution. Section III provides a brief overview on adaptive modulation schemes. The proposed approach for the optimization of the rotation angle of signal constellations, as well as the derivation of an approximate expression for the PEP of modulation diversity systems, are described in Section IV. The performance analysis of the proposed adaptive system is presented in Section V. Finally, Section VI is devoted to the conclusions and future research.

II. SYSTEM AND CHANNEL MODELS

In adaptive modulation systems, different modulation schemes are used by the transmitter, according to channel fluctuations. The modulation schemes are dynamically selected in order to maximize the spectral efficiency, under a target BER constraint [1]. The transmission rate adaptation occurs in multiples of the symbol rate. The receiver then estimates (or predicts) the channel characteristics to determine which signal constellation should be used. That information is then sent back to the sender to allow the adaptation on both sides (sender and receiver). Since the use of ideal Nyquist pulse shape is assumed, the transmitted signal bandwidth is defined as $B = 1/T_s$, in which $T_s$ is the symbol time.
In order to improve the performance of conventional adaptive modulation systems, the use of the modulation diversity technique is proposed. The modulation diversity technique mitigates the effects of the multipath fading on the transmitted signals. Figure 1 illustrates the block diagram of the proposed system.

![Block diagram of the proposed system.](image)

The fading amplitude $\alpha(t)$ is modeled as a Nakagami-$m$ stationary random variable whose probability density function (pdf) is expressed as

$$p(\alpha) = \frac{2m^m \alpha^{2m-1}}{\Gamma(m)} e^{-m\alpha^2}, \quad \alpha \geq 0,$$

where $\Gamma(\cdot)$ denotes the Gamma function, and $m \geq 1/2$ is a parameter that controls the intensity of the fading. It is assumed that the fading amplitude is perfectly estimated at the receiver, i.e., $\hat{\alpha}(t) = \alpha(t)$.

The Nakagami-$m$ distribution covers a range of multipath fading distributions, such as Semi-Gaussian, Rayleigh, Hoyt and Rice, by adjusting the parameter $m$. The value of $m$ represents the ratio between specular and diffused signal components [7].

Since slow (block) fading is assumed, the channel can be considered AWGN with a varying SNR [9]. For a Nakagami-$m$ channel, the SNR is Gamma distributed with mean channel SNR given by $\bar{\gamma} = E_s/N_0$, and $E_s$ is the symbol energy.

The fading amplitude $\alpha(t)$ is modeled by the combination of a judicious choice of the reference rotation angle $\theta$ with the independent interleaving of the symbol components. The interleaving process basically switches the imaginary parts of symbols subject to uncorrelated fading amplitude samples (i.e., symbol samples shifted in time by an interleaving delay $\kappa$). This scheme can be applied to any two-dimensional constellation used by the adaptive modulation system.

The interleaving delay ($\kappa$) should be carefully defined in order to correspond to the channel coherence time. This characteristic allows the modulation diversity system to interleave the imaginary part of the symbols subject to uncorrelated fading amplitude samples. Moreover, an important aspect to be addressed in this scenario is the selection of different modulation schemes for pairs of interleaved symbols. In this situation, the adaptive system selects different modulation schemes for the interleaved symbol blocks, increasing the average system BER (since the interleaving makes the symbol components susceptible to SNR values different from those for which they were designed).

Since this scheme requires prior knowledge of the modulation scheme to be used in the next block, a channel predictor should also be available at the receiver. The use of channel predictors for adaptive modulation is a common approach [10], [11] and this requirement does not considerably increase the system complexity. Thus, the estimated and the predicted channel parameters (SNR, $\hat{\gamma}(t)$, and fading parameter, $\hat{m}$) are used to select the appropriate modulation order $M$ and rotation angle $\theta$ for the two blocks. Those variables are then sent back to the transmitter in order to perform the adaptation of the source constellation. The feedback link is considered error free and subject to a negligible delay. Furthermore, it is assumed...
that the actual and estimated channel parameters are perfectly estimated, \( i.e., \tilde{\gamma}(t) = \gamma(t) \) and \( \hat{\gamma} = \gamma \), and they will be denoted simply by \( \gamma \) and \( \hat{\gamma} \).

One approach to deal with this problem is the dynamic adaptation of the transmit power per symbol component. In this case, some power allocation policy, such as the water-filling approach, should be applied to symbol components. Another possibility is the use of some combining strategy to select a common modulation scheme to be used in both symbols. In the proposed system a simple solution is used: the interleaver and deinterleaver subsystems are activated only if the same modulation scheme is selected for both interleaved signal blocks.

### III. ADAPTIVE MODULATION SCHEMES

In the adaptive modulation schemes, the choice of the modulation scheme is based on the definition of \( N \) decision (or fading) regions, \( R_i = [\gamma_i, \gamma_{i+1}] \), \( i = 0, \ldots, N - 1 \), in which \( \gamma_i \) is an SNR decision threshold defined to achieve some performance level (in terms of BER), with \( \gamma_N = \infty \) and \( \gamma_0 \geq 0 \). A constellation with \( M_i \) symbols (each one equivalent to \( k_i = \log_2 M_i \) bits) is used when \( \gamma(t) \in R_i \) (i.e. the instantaneous SNR value belongs to region \( i \)).

Two important performance measures of the adaptive modulation scheme are: (a) the average spectral efficiency and (b) the average BER [1]. The average spectral efficiency can be calculated as follows [12]

\[
\bar{\eta} = \sum_{i=0}^{N-1} \frac{k_i}{\gamma_i} \int p(\gamma)d\gamma.
\]

The average performance, in terms of BER, is computed using the following expression [12]

\[
\text{BER} = 1 - \frac{1}{\bar{\eta}} \sum_{i=0}^{N-1} \frac{k_i}{\gamma_i} \int \text{BER}_i(\gamma)p(\gamma)d\gamma,
\]

in which \( \text{BER}_i(\gamma) \) refers to the BER function for modulation \( i \) in AWGN channels at some received SNR \( \gamma \).

The average performance of the adaptive modulation system depends on the choice of the decision thresholds \( \gamma_i \), which should ensure that the system remains operating at or below a certain target BER. Lower threshold values lead to a high throughput. On the other hand, a high decision threshold implies a lower BER for the adaptive modulation scheme [13].

### IV. OPTIMIZATION OF THE CONSTELLATION ROTATION ANGLE

After the channel estimation and the definition of the constellation order \( M \), the proposed adaptive system should adjust the constellation rotation angle to optimize the system performance. The optimal rotation angle evaluation can be accomplished in two ways: (a) by using Monte Carlo simulation; or (b) by the optimization of the system BER expression. The first approach has a high computational cost, since many simulations must be performed for different settings and with different rotation angles. Therefore, this alternative is not feasible to be used with the system in operation.

On the other hand, obtaining an exact closed-form expression for the BER of diversity modulation systems is a difficult problem and the use of upper bounds, such as the union bound (UB) [14], is a common approach to evaluate the error probability of a two-dimensional signal constellation. Thus, assuming equiprobable symbols, the BER of the modulation diversity systems subject to Nakagami-\( m \) fading (and with the \( i \)-th modulation order) is upper bounded by [15]

\[
P_b[i] \leq F_b^\text{UB}[i] = \frac{1}{k_i^2} \sum_{s \in S_i} \sum_{k \in \mathbb{Z}} a(s, k) P(s \rightarrow k),
\]

in which \( S_i \) represents the \( i \)-th signal constellation with \( M_i \) signals, \( k_i = \log_2 M_i \) is the number of bits per symbol of the constellation, \( a(s, k) \) is the Hamming distance between the codewords associated to \( s \) and \( k \) and \( P(s \rightarrow k) \) is the PEP (Pairwise Error Probability) that \( k \) is estimated by the receiver when \( s \) has been transmitted.

Since interleaving is employed in the transmitted symbols, the I and Q symbol components are subject to independent fading. Let \( \alpha_I \) and \( \alpha_Q \) be the Nakagami-\( m \) distributed fading amplitudes in the I and Q channels, respectively. Thus, the PEP for a system with modulation diversity, subject to Nakagami-\( m \) fading, is given by [14]

\[
P(s \rightarrow k) = \frac{4m^{2m}}{\Gamma^2(m)} \int_0^\infty \int_0^{2m-1} e^{-m(a_I^2 + a_Q^2)} P_{\alpha_I}(\gamma_I) P_{\alpha_Q}(\gamma_Q) \, d\alpha_I d\alpha_Q,
\]

in which \( d_{\alpha_I}^2 \) and \( d_{\alpha_Q}^2 \) are the Euclidean distances between the symbols \( s \) and \( k \) in the I and Q components, respectively, and \( \tilde{\gamma} \) is the channel mean SNR.

An exact expression for the PEP, based on the Craig’s formula [16] for the \( Q(\cdot) \) function (the Gaussian tail function), is proposed in [5]. The exact formula is given by

\[
P(s \rightarrow k) = \frac{2 \pi(1 + c_I)^m (1 + c_Q)^m}{2m} F_1 \left( \frac{1}{2}, m, m, 2m + 1, \frac{1}{1 + c_I}, \frac{1}{1 + c_Q} \right),
\]

in which \( c_I = \frac{\gamma_I^2}{4m} \), \( c_Q = \frac{\gamma_Q^2}{4m} \), \( F_1(a, y) \) is the Appell hypergeometric function [17] and \( B(\cdot, \cdot) \) is the Beta function [17].

The exact PEP expression is too complex to be used for real time optimization. The authors of [5] have proposed an approximation for (9) considering high SNR values, but the approximation also contains a Beta function, making its use impractical for the proposed system.

In this context, a new PEP approximation for Nakagami-\( m \) channels was derived. Based on the results presented in [18, Eq. (12)], the \( Q(\cdot) \) function was approximated by

\[
Q(x) \approx \frac{1}{12} e^{-x^2/2} + \frac{1}{4} e^{-2x^2/3}, \quad x \geq 0.
\]

Substituting (10) into (8), performing the integration and the analytical manipulations, the approximated exponential PEP expression can be written as

\[
P(s \rightarrow k) \approx \frac{1}{12} \left[ 1 + \frac{\gamma_I^2}{4m} \right]^{-m} + \frac{1}{4} \left[ 1 + \frac{\gamma_Q^2}{3m} \right]^{-m}. 
\]
In the golden search method an angle for a QPSK system and different values of the parameter \( \theta \) is confirmed by those presented in [5, Table 1] (by small differences on precision).

As can be seen in Figure 2, the PEP expression represents a sufficiently accurate approximation for the BER of modulation diversity systems. The expression is simple and adequate for a precise evaluation of the \( \theta \) angle. Furthermore, the precision of the PEP approximation can be improved (at the cost of a higher complexity in the final expression) if additional exponential terms are introduced in (10).

The PEP approximation allows the optimization of the \( \theta \) angle, and an efficient optimization technique is the golden section search [19], a technique for finding an extremum value inside an interval. This technique permits a selection of \( \theta \) that minimizes the overall system BER with a relatively small number of recursions. In the golden search method an interval is continuously divided (according to the so-called golden ratio) until the optimum \( \theta \) value is found with a given precision.

Table I presents some exact values for the optimum rotation angle \( \theta \). Since the optimum value of \( \theta \) converges at high SNR values, an SNR of 40 dB was adopted in the optimization process. Furthermore, the results obtained for QPSK systems are confirmed by those presented in [5, Table 1] (by small differences on precision).

It is important to note that, the occurrence of outage was not considered in the simulation. Thus, even during the low SNR channel states, the system does not abort the transmissions. This can be a desirable characteristic in some systems, since some network protocols or applications can be sensitive to the communication latency. However, the absence of outage in the system makes the average BER exceeds the target BER.

Figures 3(a) and 3(b) present the performance of the proposed (with modulation diversity) and conventional (without modulation diversity) adaptive modulation systems in terms of average BER and spectral efficiency, respectively. Both theoretical (computed with (5) and (6)) and simulated values are presented considering a channel subject to Nakagami-\( m \) channel (parameter \( m = 2 \)).

As can be seen in Figure 3(a), the proposed system (“Mod. Div.”) shows a noticeable gain in terms of the average system BER for high SNR values. For the “Profile 1”, the proposed system achieved, approximately, a gain of 3.79 dB for a BER value of 1.85 \( \times 10^{-5} \) relative to a system without the modulation diversity. In addition, a gain of 3.44 dB was obtained for a BER value of 3.24 \( \times 10^{-6} \) using the “Profile 2”.

The system average spectral efficiency shows no loss, as can be confirmed in Figure 3(b), because the proposed system did not change the modulation schemes selected by the adaptive modulator. Instead, the modulation diversity was only used in pairs of symbol blocks in which the same modulation scheme has been selected. Therefore, the interleaving, combined with the dynamic rotation of the signal constellations (based on the approximation presented in (11)), has improved the adaptive modulation system by reducing the average system BER in high SNR values.

V. Performance Analysis and Results

This section presents the performance analysis of an adaptive modulation system that uses modulation diversity. Numerical evaluations and Monte Carlo simulations were performed to verify the gain obtained by the proposed system. Python language and the Mpmath library were used in the numerical evaluation, while Monte Carlo simulations were implemented in C language. The frame length (384 symbols) and the modulation schemes (QPSK, 16-QAM and 64-QAM) adopted in the simulations were based on the release 7 of the UMTS/HSDPA protocol [20]. No channel coding was used in the evaluation.

The experiments used a Nakagami-\( m \) channel model with \( m = 2 \). Two different performance profiles were designed for the experiments, each one associated with a target BER \((10^{-2} \text{ and } 10^{-3})\). Based on the target BER of each profile, appropriate decision regions \( R_i \) were obtained by a numerical inversion of the adopted AWGN BER functions. The decision thresholds are presented in Table II.

![Figure 2. Exponential and exact union bounds for a QPSK system with modulation diversity considering different rotation angles. This evaluation was performed considering a Nakagami-\( m \) fading channel (\( m = 2.0 \) with SNR= 14 dB (optimum rotation angle: 29.48°)).](image)

![Figure 2](image)

**TABLE I**

<table>
<thead>
<tr>
<th>QPSK</th>
<th>( m )</th>
<th>0.5</th>
<th>1.0</th>
<th>2.0</th>
<th>4.0</th>
<th>8.0</th>
<th>16.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \theta_{opt} )</td>
<td>28.90°</td>
<td>29.63°</td>
<td>30.36°</td>
<td>30.92°</td>
<td>31.31°</td>
<td>31.57°</td>
<td></td>
</tr>
<tr>
<td>16-QAM</td>
<td>( m )</td>
<td>0.5</td>
<td>1.0</td>
<td>2.0</td>
<td>4.0</td>
<td>8.0</td>
<td>16.0</td>
</tr>
<tr>
<td>( \theta_{opt} )</td>
<td>21.60°</td>
<td>21.62°</td>
<td>21.77°</td>
<td>22.18°</td>
<td>31.75°</td>
<td>31.78°</td>
<td></td>
</tr>
</tbody>
</table>

VI. Conclusion and Future Work

This paper presented the architecture and performance evaluation of an improved adaptive modulation scheme based on modulation diversity. The data rate is dynamically modified (by changing the signal constellation order) to ensure a maximum average system BER.

Since the optimal rotation angle depends on the chosen modulation order and fading intensity, the constellation rotation...
The proposed solution is the dynamic activation of the interleaver at SNR values different from those for which they were designed. Otherwise, the interleaver is deactivated.

An important issue addressed by the proposed system is the divergence of modulation schemes in pairs of interleaved symbols. In this situation, the use of modulation diversity increases the average system BER, since the process produces symbols with components modulated using different constellation orders and, therefore, susceptible to SNR values different from those for which they were designed. The proposed solution is the dynamic activation of the interleaver, if the interleaved symbol blocks use the same modulation order. Otherwise, the interleaver is deactivated.

A performance analysis considering a system with parameters similar to the UMTS/HSDPA protocol was performed. The evaluation of the proposed system has shown that significant gains (of about 4 dB), in terms of the average system BER, are achieved for high SNR values. Smaller gains were also obtained in the central region of the BER curve (between 5 and 20 dB). Furthermore, the use of modulation diversity did not affect the proposed system spectral efficiency, considering a conventional adaptive modulation scheme with the same configuration.

Future research includes the evaluation of other solutions for the interleaving of symbols with different modulation orders. As possible solutions one will investigate the use of a variable power control per symbol component and the use of different combining strategies to select a common modulation order that minimizes the loss and maximizes the data rate for the interleaved symbols.

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**REFERENCES**